STUDIES AND EXERCISES

IN

FORMAL LOGIC.

No. 17. p.

STUDIES AND EXERCISES

IN

FORMAL LOGIC,

INCLUDING

A GENERALISATION OF LOGICAL PROCESSES IN THEIR
APPLICATION TO COMPLEX INFERENCES,

BY

JOHN NEVILLE KEYNES, M.A.,

LATE FELLOW OF PEMBROKE COLLEGE, CAMBRIDGE.

784
1/4/1890

PREFACE.

IN addition to problems worked out in detail and unsolved problems, by means of which the student may test his command over logical processes, the following pages contain a somewhat detailed exposition of certain portions of what may be called the book-work of Formal Logic. This was necessary in the case of disputed or doubtful points in order that the working out of the problems might be made consistent and intelligible; there were also some points concerning which I was dissatisfied with the method of treatment adopted in the ordinary text-books. At the same time, this volume must be regarded, not as superseding the study of an elementary text-book of Formal Logic, but as supplementing it. While certain topics are dealt with in considerable detail others have been omitted; *e.g.*, the doctrines of Definition and Division and the Predicables are not touched upon, no definition of the Science itself is given, and no systematic discussion of first principles has been

introduced. For a general outline of my views on the position of Formal Logic I may refer the reader to an article in *Mind* for July, 1879. For several reasons I should have been glad to rewrite and in some respects to modify this paper; but anything like an adequate treatment of the subject would have enlarged the book considerably beyond the limits that I had assigned to it.

I have not endeavoured to distinguish definitely between book-work and problem; and the unanswered exercises are not separated and placed apart at the end of the chapters, but are introduced at the points at which the student who is systematically working through the book will find himself in a position to solve them. Exercises of a similar character have not been to any considerable extent multiplied, but I believe that no kind of problem relating to the operations of Formal Logic has been overlooked. By reference to sections 261, 262, 281—285, the reader will find that the ordinary syllogism admits of problems of some complexity.

In the expository portions of Parts I. II. and III., dealing respectively with Terms, Propositions, and Syllogisms, I have in the main followed the traditional lines, though with a few modifications; *e. g.*, in the systematization of immediate inferences, and in some points of detail in connection with the syllogism to which I need not make further reference here. For purposes of illustration Euler's diagrams are em-

ployed to a greater extent than is usual in English manuals.

In Part IV., which contains a generalisation of logical processes in their application to complex inferences, a somewhat new departure is taken. So far as I am aware this part constitutes the first systematic attempt that has been made to deal with formal reasonings of the most complicated character without the aid of mathematical symbols and without abandoning the ordinary non-equational or predicative form of proposition. In this attempt I have met with greater success than I had anticipated; and I believe that the methods which I have formulated will be found to be as easy of application and as certain in obtaining results as the mathematical, symbolical, or diagrammatic methods of Boole, Jevons, Venn and others. The reader may judge of this for himself by comparing with Boole's own solutions the problems discussed in sections 368, 369, 383—386; or by solving by different methods other of the problems, e. g., the very complex one contained in section 408. The book concludes with a general method of solution of what Professor Jevons called the Inverse Problem, and which he himself seemed to regard as soluble only by a series of guesses.

Of the Questions and Problems more than half are my own composition. Of the remainder, about a hundred have been taken from various exami-

nation papers, and about sixty are from the published
writings of Boole, De Morgan, Jevons, Solly, Venn and
Whately. In the latter case the name of the author
is appended, generally with a reference to the work
from which the example is taken. In the case of
problems selected from examination papers, a letter
is added indicating their source, as follows:—C.=
University of Cambridge ; L.=University of London ;
N. = J. S. Nicholson, Professor of Political Economy
in the University of Edinburgh ; O. = University of
Oxford ; R.=G. Croom Robertson, Professor of Mental
Philosophy and Logic in University College, London;
V.= J. Venn, Fellow and Lecturer of Gonville and
Caius College, Cambridge; W.= J. Ward, Fellow and
Assistant Tutor of Trinity College, Cambridge.

The logicians to whom I have been chiefly in-
debted are De Morgan, Jevons and Venn. De Mor-
gan's various logical writings are rendered somewhat
formidable and uninviting by reason of the multipli-
cation of symbols and formulae which he is never
tired of introducing, and this is probably the reason
why they are little read at the present time ; they
nevertheless constitute a mine of wealth for all who
are interested in the developments of Formal Logic.
With Jevons I have continually found myself in dis-
agreement on points of detail, and it is possible that I
may give the impression of having taken up a special
position of antagonism with regard to him. This is
far from being really the case. I believe that since

Mill no one else has given such an impetus to the study of Logic, and I hold that in more than one direction he has led the way in new developments of the science that are of great importance.

To Mr Venn I am peculiarly indebted, not merely by reason of his published writings, especially his Symbolic Logic, but also for most valuable suggestions and criticisms given to me while this book was in progress. I am glad to have this opportunity of expressing to him my thanks for the ungrudging help he has afforded me. I am also under great obligation to Miss Martin of Newnham College and to Mr Caldecott of St John's College for criticisms which I have found very helpful.

CONTENTS.

PART I.

TERMS.

PART II.

PROPOSITIONS.

PART III.

SYLLOGISMS.

PART IV.

A GENERALISATION OF LOGICAL PROCESSES IN THEIR APPLICATION TO COMPLEX PROPOSITIONS.

STUDIES AND EXERCISES IN FORMAL LOGIC.

PART I.

TERMS.

CHAPTER I.

GENERAL AND SINGULAR NAMES. CONCRETE AND ABSTRACT NAMES.

1. Brief definitions of *word, name, term, symbol, concept.*

A *word* is an articulate sound, or the written equivalent of an articulate sound, which either by itself or in conjunction with other words, constitutes a name, or forms a sentence.

A *name* is " a word taken at pleasure to serve for a mark which may raise in our mind a thought like to some thought we had before, and which being pronounced to others, may be to them a sign of what thought the speaker had or had not before in his mind." *Hobbes.*

A *term* is a name regarded as the subject or the predicate of a proposition.

A *symbol*, in its widest signification, is a sign of any kind ; narrowing our point of view, it is any written sign ; and narrowing it still more, it is a written sign which is employed without the realization at each step of its full signification. Thus, when symbols are used in algebraical reasoning, it is according to certain fixed rules, without reference to or thought of their ulterior meaning. Names may themselves be employed as symbols in this sense. Of course, in the widest sense, all names are symbols.

A *concept* is defined by Sir William Hamilton as "the cognition or idea of the general character or characters, point or points, in which a plurality of objects coincide." In other words, a concept is the mental equivalent of a general name.

2. Categorematic and Syncategorematic Words.

A *categorematic* word is one which can by itself be used as a term, *i. e.*, which can stand alone as the subject or the predicate of a proposition.

A *syncategorematic* word is one which cannot by itself be used as a term, but only in combination with one or more other words.

Any noun substantive in the nominative case, or any other part of speech employed as equivalent to a noun substantive, may be used categorematically.

Adjectives are sometimes said to be used categorematically by a *grammatical ellipsis*. In the examples, "The rich are happy," "Blue is an agreeable colour," either a substantive is understood as being qualified by the adjective, or the adjective is used as a substantive, that is, as a mark of something, not merely as a mark qualifying something.

Any part of speech, or the inflected cases of nouns

substantive, may be used categorematically by a *suppositio materialis*, that is, by speaking of the mere word itself as a thing; for example, "John's is a possessive case," "Rich is an adjective," "With is an English word."

Using the word *term* in the sense in which it was defined in the preceding section, it is clear that we ought not to speak of syncategorematic *terms*.

3. *General, singular* and *proper* names.

A *general* name is a name which is capable of being truly affirmed, in the same sense, of each of an indefinite number of things, real or imaginary. A *singular* name is a name which is capable of being truly affirmed, in the same sense, of only one thing, real or imaginary. A *proper* name is a singular name given merely to distinguish an individual person or thing from others, its application after it has been once given being independent of any special attributes that the individual may possess[1].

Thus, *Prime Minister of England* is a general name, since at different times it may be applied to different individuals. We may, for example, talk about "the prime ministers of England of the present century." The name is however made singular by the prefix "*the*," meaning "the present prime minister," or "the prime minister at the time to which we are referring." Similarly any general name may be made singular; for example, man, *the first man;* star, *the pole star.*

The name *God* is singular to a monotheist as the name of the Deity, general to a polytheist, or as the name of anything worshipped by anybody. *Universe* is

---

[1] A proper name might perhaps be defined as "a non-connotative singular name." But this definition presupposes a distinction which is best given subsequently, and it would give rise to a controversy, that also had better be postponed. Compare section 14.

general in so far as we distinguish different kinds of universes, *e.g.*, the material universe, the terrestrial universe, &c.; it is singular if we mean *the* universe. *Space* is general if we mean a particular portion of space, singular if we mean space in the aggregate. *Water* is general. Professor Bain takes a different view here; he says, " Names of Material—earth, stone, salt, mercury, water, flame,—are singular. They each denote the entire collection of one species of material" (*Logic, Deduction*, pp. 48, 49). But when we predicate anything of these terms it is generally of *any portion* (or of some particular portion) of the material in question, and not of the entire collection of it *considered as one aggregate;* thus, if we say, "Water is composed of oxygen and hydrogen," we mean any and every particle of water, and the name has all the distinctive characters of the general name. Similarly with regard to the other terms mentioned in the above quotation. It is also to be observed that we distinguish different kinds of stone, salt, &c.

A name is to be regarded as general if it may be *potentially* affirmed of more than one, although it accidentally happens that as a matter of fact it can be actually affirmed of only one, *e.g.*, *King of England and Spain.* We must also note the case in which we are dealing with a name that actually is not applicable to any individual at all; *e.g.*, *President of the British Republic.* A really singular name is distinguished from these by not being even potentially applicable to more than one individual; *e.g.*, *the last of the Mohicans, the eldest son of King Edward the First*[1].

---

[1] It seems desirable to make the distinction implied in this paragraph; still I am not sure that it might not in some cases be very difficult to apply it satisfactorily. Nearly all these divisions of names tend to give rise in the last resort to metaphysical difficulties; but, in my opinion, these should as far as possible be avoided in a logical treatise.

Victoria is the name of more than one individual, and
can therefore be truly affirmed of more than one individual.
Is it therefore general? Mill answers this question in the
negative, and rightly, on the ground that the name is not
here affirmed of the different individuals in the same sense.
Professor Bain brings out this distinction very clearly in his
definition of a general name: "A general name is applicable
to a number of things in virtue of their being similar, or
having something in common." Victoria is then not general
but singular; and it belongs to the sub-class of proper
names.

4. Collective Names; and the collective use of
names. Are all collective names singular?

A *collective* name is one which is the name of a group
of things considered as one whole; *e.g.*, regiment, nation,
army.

A collective name may be singular or general. It is
the name of a group or collection of things, and so far
as it is capable of being truly affirmed in the same sense
of only one such group, it is singular; *e.g.*, the 29th regi-
ment of foot, the English nation, the Bodleian library. But
if it is capable of being truly affirmed in the same sense of
each of several such groups it is to be regarded as general;
e.g., regiment, nation, library. Professor Bain writes as if a
name could be general and singular at the same time,—
"Collective names as nation, army, multitude, assembly,
universe, are singular; they are plurality combined into
unity. But, inasmuch as there are many nations, armies,
assemblies, the names are also general. There being but
one 'universe', that term is collective and singular". I should
rather say that as the above stand, with the possible excep-
tion of universe, they are not singular at all. Mill and

others imply that there is a distinction between collective
and general names. The real distinction however is be-
tween the *collective* and *distributive* use of names. A col-
lective name such as nation, or any name in the plural
number, is the name of a collection or group of things.
These we may regard as one whole, and something may
be predicated of them that is true of them only as a
whole ; in this case the name is used *collectively*. On the
other hand, the group may be regarded as a series of units,
and something may be predicated of these which is true
of them only taken individually ; in this case the name
is used *distributively*. Also, when anything is predicated
of a series of such groups the name is used distributively.

The above distinction may be illustrated by the pro-
positions,—All the angles of a triangle are equal to two
right angles, All the angles of a triangle are less than two
right angles. The subject term is the same in both these
cases, but in the first case the predication is true only of
the angles all taken together, while in the second it is true
only of each of them taken separately ; in the first case
therefore the term is used collectively, in the second dis-
tributively.

The peculiarity, then, of a collective name is that it can
be used collectively in the singular number, while other
names can be used collectively only in the plural number ;
compare, for example, the names 'clergyman' and 'the
Clergy.' Collective names in the plural number may them-
selves be used distributively, and it is therefore not correct
to say that all collective names are singular. It may indeed
be held that, while this is true, still when a name is used
collectively, it is equivalent to a singular name. For
example, The whole army was annihilated, The mob filled
the square. But I am doubtful whether even this is true in

such a case as the following,—In all cases all the angles
of a triangle are equal to two right angles.

5. Select the terms that are used collectively in
the following propositions; also classify the terms
contained in these propositions according as they
are collective, singular, and general respectively, and
find in what way these classes overlap one another:—

The Conservatives are in the majority in the
House of Lords.

All the tribes combined.

The nations of the earth rejoiced.

Crowds filled all the churches.

One generation passeth away and another genera-
tion cometh.

Your boxes weigh 140 lbs.

The volunteers mustered in considerable numbers.

Time flies.

True poets are rare.

Those who succeeded were few in number.

The mob was soon dispersed.

Our armies swore terribly in Flanders.

The multitude is always in the wrong.

6. Abstract and Concrete Names.

Mill defines abstract and concrete names as follows :—
" A concrete name is a name which stands for a thing ;
an abstract name is a name which stands for an attribute
of a thing" (*Logic*, I. p. 29)[1]. In many cases, this distinction
is of easy application; for example, *triangle* is the name of
something that possesses the attribute of being bounded by

---

[1] The references are to the ninth edition of Mill's *Logic*.

three straight lines, and it is a concrete name; *triangularity* is the name of this distinctive attribute of triangles, and it is an abstract name. But there are other cases to which the application of the distinction is difficult; and an attempt at more precise definition is liable to involve us in metaphysical discussions such as the logician should if possible avoid. The first question that arises is what precisely is meant by the word *thing*, when it is said that a concrete name is the name of a thing. By a thing, we may mean anything that exists; but we cannot mean that here, since "attributes" exist, and the distinction between concrete and abstract names would vanish. Again, by a thing we may mean a substance; but *substances* are contrasted with *feelings* as well as with *attributes*, and this threefold division would make names of feelings neither abstract nor concrete, which can hardly be intended. With regard to the proper place of names of states of consciousness it would be generally agreed to call them concrete. Thus, while *sensibility*, the faculty of experiencing sensation, is an abstract name, the name of a sensation itself should be regarded as concrete, being the name of something which possesses attributes, for example, of being pleasurable or painful, of being a sensation of touch or one of hearing. But here again a difficulty arises, since, as pointed out by Mill, in many cases "feelings have no other name than that of the attribute which is grounded on them." For example, by *colour* we may mean sensations of blue, red, green, &c., or we may mean the attribute which all coloured objects possess in common. In the former case, colour is a concrete name, in the latter an abstract name. *Sound*, again, is concrete, in so far as it is the name of a sensation, *e.g.,* "the same sound is in my ears which in those days I heard"; but in the following cases, it should rather be

regarded as abstract,—"a tale full of sound and fury," "a name harsh in sound."

The matter is still further complicated if Mill's view is taken, and attributes are analysed into sensations, "the distinction which we verbally make between the properties of things and the sensations we receive from them, originating in the convenience of discourse rather than in the nature of what is signified by the terms." For logical purposes however we certainly need not pursue the analysis so far as this.

But still another difficulty arises from the fact that we sometimes speak of attributes themselves as having attributes; and so far as this is permissible, we must agree with Professor Jevons that "abstractness becomes a question of degree." It may be said that *civilization* is abstract regarded as an attribute of a given state of society, but that it becomes concrete regarded as itself possessing the attribute of progressiveness or the attribute of stationariness[1].

Besides all the above, we have to notice that terms originally abstract are very liable to come to be used as concrete, and this may create further confusion. Thus, Professor Jevons remarks,—"*Relation* properly is the abstract name for the position of two people or things to each other, and those people are properly called *relatives*. But we constantly speak now of *relations*, meaning the persons themselves; and when we want to indicate the abstract relation they have to each other we have to invent a new abstract name *relationship*. Nation has long been a con-

---

[1] It does not however follow that we should regard the name of a complex attribute as therefore concrete. *Civilization* regarded as possessing the attribute of stationariness may be considered concrete, while *stationary civilization* regarded as the attribute of a given state of society may still be considered abstract.

crete term, though from its form it was probably abstract
at first; but so far does the abuse of language now go,
especially in newspaper writing, that we hear of *a nationality*,
meaning a nation, although of course if nation is the con-
crete, nationality ought to be the abstract, meaning the
quality of being a nation.   Similarly, *action, intention, ex-
tension, conception*, and a multitude of other properly abstract
names, are used confusedly for the corresponding concrete,
namely, *act, intent, extent, concept*, &c." (*Elementary Lessons
in Logic*, pp. 21, 22).

The outcome of the whole discussion seems to be that if
we are asked whether a given name is abstract or concrete,
we frequently cannot give an absolute answer, but have to
distinguish between different cases.   Given any two terms
however which are connected together, we can undertake
to say which of them, if either, is abstract in relation to
the other.

7.   How would you apply the distinction between
abstract and concrete names to the following:—
*life, fate, logic, time, fault, generosity, the habit of
talking loudly?*

8.   So far as you can, name the concrete terms
corresponding to such of the following as you regard
as abstract, and the abstract terms corresponding to
such of them as you regard as concrete:—*antithesis,
Socrates, attempt, equation, yellow, richness, resentment,
temper, angel, charity, bounty, compassion, mercy.*

9.   Can the distinction between *singular* and
general be applied to abstract names?

This question is sometimes answered as follows :—Most
abstract names are general, because they are names of

attributes which are found in different objects. Deity, however, to the monotheist, may be given as an example of a singular abstract, since it is the name of an attribute which can be affirmed of God only.

This criterion would make the corresponding abstract of every general concrete name, general, and of every singular concrete name, singular ; but it is evidently based on a fundamental confusion. By an abstract name we mean the name of an attribute considered apart from the things possessing that attribute ; and the attribute is to be regarded as one and the same whether it is possessed by one thing only, or by an indefinite number of things.

Mill takes another ground of distinction. He says, "Some abstract names are certainly general. I mean those which are names not of one single and definite attribute, but of a class of attributes. Such is the word *colour*, which is a name common to whiteness, redness, &c. Such is even the word whiteness, in respect of the various shades of whiteness to which it is applied in common ; the word magnitude, in respect of the various degrees of magnitude and the various dimensions of space ; the word weight, in respect of the various degrees of weight. Such also is the word *attribute* itself, the common name of all particular attributes. But when only one attribute, neither variable in degree nor in kind, is designated by the name ; as visibleness ; tangibleness ; equality ; squareness ; milk-whiteness ; then the name can hardly be considered general; for though it denotes an attribute of many different objects, the attribute itself is always conceived as one not many" (*Logic*, I. p. 30). I should doubt if any attribute can, strictly speaking, be conceived as *many*. An attribute in itself is one and indivisible, and does not admit of numerical distinction. When we begin to distinguish kinds and differences, which

we can only do by the addition of other attributes, the name would appear to begin to partake of the concrete character. I should therefore doubt the propriety of saying that some abstract names are certainly general. It would be more appropriate to call all strictly abstract names singular. A still more satisfactory solution however is to consider the distinction of general and singular as not applying to abstract names at all. Mill himself indicates this view, remarking that, "to avoid needless logomachies, the best course would probably be to consider these names as neither general nor individual, and to place them in a class apart" (*Logic*, I. p. 30).

10. Do abstract terms admit of being put in the plural number? Distinguish between the terms which are abstract and concrete in the following list, and at the same time indicate which can in your opinion be used in the plural:—*colour, redness, weight, value, quinine, equation, heat, warmth, hotness, solitude, whiteness, paper, space, gold.* [C.]

11. The Connotation and Denotation of Terms.

Every concrete general name is the name of a class, real or imaginary: by its *connotation* we mean the attributes on account of which we place any individual in the class or call it by the name; by its *denotation* we mean the individuals which possess these attributes, and which are therefore placed in the class and called by the name. The terms *intension* (or *comprehension*), and *extension* are also used as equivalent to connotation and denotation respectively. Strictly speaking these terms belong to the Conceptualist Logic, and should be applied to concepts rather than to names.

Thus, the connotation of " plane triangle " is given when it is defined as a plane figure contained by three straight lines; under its denotation are included all plane figures fulfilling this condition. The connotation of "man" consists of those attributes, whatever they may be, which we regard as essential to the class man, *i.e.*, in the absence of any one of which we should refuse to call any individual

by the name; its denotation is made up of all the individuals actually possessing these attributes.

12. Mill's use of the term *connotative* compared with that of other writers.

(i) "A non-connotative term is one which signifies a subject only, or an attribute only. A connotative term is one which denotes a subject, and implies an attribute" (Mill, *Logic*, I. p. 31). According to this definition, a connotative name must possess *both* connotation and denotation.

The following kinds of names are connotative in Mill's sense :—(1) All concrete general names. (2) Some singular names. For example, "city" is a general name, and as such no one would deny it to be connotative. Now if we say "the largest city in the world", we have individualised .the name, but it does not thereby cease to be connotative. Proper names are, however, according to Mill, not connotative, since they merely denote a subject and do not imply any attributes. To this point, which is a disputed one, we must return. Bain (*Logic, Deduction*, p. 49) implies that only general names are connotative; but this can hardly have been intended. (3) Most abstract names are non-connotative, since they merely signify an attribute and do not denote a subject. Mill however maintains that some abstract names are connotative, namely, the names of attributes that may have attributes ascribed to them. To this point also we must return.

(ii). The use of the word "connotative" does not seem to have been quite fixed with the schoolmen. Mansel (*Aldrich*, p. 17), while admitting that there was some license in the use of the word, gives the following account on the authority of Occam. With the schoolmen, a connotative term was one that "primarily signified an attribute, second-

arily a subject;" (and it was said to *connote* or *signify secondarily* the subject). Thus "white" was regarded as connotative, whilst the original substances or attributes, as "man" or "whiteness" were called *absolute;* the former signifying primarily a subject, the latter not signifying a subject at all. Only adjectives and participles therefore (words called by Professor Fowler "attributives") are connotative in this sense.

Mill (*Logic*, I. p. 42, note) says that the schoolmen used it in his own sense, though some of their expressions are vague. He quotes James Mill as using it more nearly in the sense ascribed by Mansel to the schoolmen.

(iii) Professor Fowler uses the term connotative in a sense different from that of Mill. "A term may be said to *denote* or designate individuals or groups of individuals, to *connote* or mean attributes or groups of attributes." In this sense, general names are both connotative and denotative; abstract names are connotative but not denotative[1], (whereas, according to Mill, they are generally speaking denotative but not connotative). This use of the term avoids some difficulties, and I am inclined to regard it as preferable to Mill's. Indeed Mill himself seems to suggest it in one place. He says that James Mill "describes abstract names as being properly concrete names with their connotation dropped: whereas, in his own view, it is the *de*notation which would be said to be dropped, what was previously connoted becoming the whole signification" (*Logic*, I. p. 42 note).

As far as we can I think we should speak merely of the "denotation" and "connotation" of names, rather than of "denotative" and "connotative names".

---

[1] Fowler, *Deductive Logic*, p. 19.

13. Is every property possessed by a class connoted by the class-name?

Unfortunately we do not find complete agreement among logicians with regard to the answer that should be given to this question; and I am inclined to think that in discussing points connected with "connotation" writers sometimes misunderstand each other, because they do not apprehend that there is fundamental disagreement between them upon this point.

I will first give Mill's answer to the question, an answer with which I should myself concur.

By the connotation of a class-name he does not mean *all* the properties that may be possessed in common by the class, but only those on account of the possession of which any individual is placed in the class, or called by the name. In other words, we include in the connotation of a class-name only those attributes upon which the classification is founded, and in the absence of any of which we should not regard the name as applicable. For example, although all equilateral triangles are equiangular we should not include equiangularity in the connotation of equilateral triangle; although all kangaroos may happen to be *Australian* kangaroos, this is not part of what we mean to *imply* when we use the name,—an animal subsequently found in the interior of New Guinea, but otherwise possessing all the properties of kangaroos would not have the name kangaroo denied to it; although all ruminant animals are cloven-hoofed, we cannot regard cloven-hoofed as part of the *meaning* of ruminant, and we may say with Mill that were an animal to be discovered which chews the cud, but has its feet undivided, it would certainly still be called ruminant.

The above meaning of connotation is that to which in

my opinion we should strictly adhere. It is of course open to any one to say that he will include in the connotation of a class name *all* the properties possessed in common by all members of the class; but this is simply to use the term *in a different sense*. It is used in this sense by a writer in a recent number of *Mind*. " On the connotative side a name means, *to us*, all those qualities common to the class named *with which we are acquainted;*—all those properties that are said to be ' involved in our idea' of the thing named. These are the properties that we ascribe to an object when we call it by the name. But, just as the word 'man,' for example, denotes every creature, or class of creatures having the attributes of humanity, whether we know him or not, so does the word properly connote the *whole* of the properties common to the class, whether we know them or not. Many of the facts, known to physiologists and anatomists about the constitution of man's brain, for example, are not involved in most men's idea of the brain : the possession of a brain precisely so constituted does not, therefore, form any part of their meaning of the word 'man.' Yet surely this is properly connoted by the word" (E. C. Benecke, in *Mind*, 1881, p. 532). Professor Jevons also uses the term in the same sense. " A term taken in intent (connotation) has for its meaning the whole infinite series of qualities and circumstances which a thing possesses. Of these qualities or circumstances some may be known and form the description or definition of the meaning; the infinite remainder are unknown" (*Pure Logic*, p. 4). Professor Bain appears to use the term in an intermediate sense, including in the connotation of a class-name not *all* the attributes common to the class but all the *independent* attributes, that is, all that cannot be derived or inferred from others.

It ought to be made very clear in any discussion con-

cerning the connotation of names in which of these several
senses we are using the term " connotation " itself.

It may be said that to use the term in Mill's sense, and
to make connotation depend on what is intended to be
implied by the mere use of the name, is to make it vary
with every different speaker. By the same name two people
may mean to imply different things, that is, the attributes
they would include in the connotation of the name would
be different; and not unfrequently some of us may be
unable to say precisely what is the meaning that we our-
selves attach to the words we use. This is a fact which it
is most important to recognise. But for the purposes of
formal logic we may assume that every name has a fixed
and definite connotation. The object of the definition of
names already in use is just to give this; and in the case
of an ideal language properly employed every name would
have the same fixed and precise meaning for everybody.

14. Are proper names connotative or non-
connotative ?

On the question here raised Mill speaks decisively,—
" The only names of objects which connote nothing are
proper names; and these have, strictly speaking, no signifi-
cation " (*Logic*, I. p. 36); and most logicians are in agree-
ment with him. An opposite view is however taken by
Jevons, and some others (*e.g.*, F. H. Bradley, T. Shedden).

In one or two places I am inclined to think that Jevons
tends somewhat to obscure the point at issue. Thus with
reference to Mill he says,—" Logicians have erroneously
asserted, as it seems to me, that singular terms are devoid
of meaning in intension, the fact being that they exceed all
other terms in that kind of meaning " (*Principles of Science*,
I. pp. 32, 33, with a reference to Mill in the foot-note).

But Mill distinctly says that some singular names are connotative, *e.g.*, the sun, the first emperor of Rome (*Logic*, I. pp. 34, 5). Again, Jevons says,—"There would be an impossible breach of continuity in supposing that after narrowing the extension of 'thing' successively down to animal, vertebrate, mammalian, man, Englishman, educated at Cambridge, mathematician, great logician, and so forth, thus increasing the intension all the time, the single remaining step of adding Augustus de Morgan, Professor in University College, London, could remove all the connotation, instead of increasing it to the utmost point" (*Studies in Deductive Logic*, pp. 2, 3). But every one would allow that we may narrow down the extension of a term till it becomes individualised without destroying its intension or connotation ; "the present Professor of Pure Mathematics in University College, London" is a singular term,—we cannot diminish the extension any further,—but it is certainly connotative.

We must then clearly understand that the only controversy is with regard to what are strictly *proper* names. Even yet there is a possible source of ambiguity that should be cleared up. If by the connotation of a name we mean *all* the attributes possessed by the individuals denoted by the name, or even all the independent attributes, Professor Jevons's view may be correct. This does appear to be what Jevons himself means, but it is distinctly *not* what Mill means,—he means only those attributes which are implied by the name itself. Jevons puts his case as follows :—"Any proper name, such as John Smith, is almost without meaning until we know the John Smith in question. It is true that the name alone connotes the fact that he is a Teuton, and is a male ; but, so soon as we know the exact individual it denotes, the name surely implies, also,

the peculiar features, form, and character, of that individual. In fact, as it is only by the peculiar qualities, features, or circumstances of a thing, that we can ever recognise it, no name could have any fixed meaning unless we attached to it, mentally at least, such a definition of the kind of thing denoted by it, that we should know whether any given thing was denoted by it or not. If the name John Smith does not suggest to my mind the qualities of John Smith, how shall I know him when I meet him? for he certainly does not bear his name written upon his brow " (*Elementary Lessons in Logic*, p. 43). A wrong criterion of connotation in Mill's sense is here taken. The connotation of a name is not the quality or qualities by which I or any one else may happen to recognise the class which it denotes. For example, I may recognise an Englishman abroad by the cut of his clothes, or a Frenchman by his pronunciation, or a proctor by his bands, or a barrister by his wig; but I do not *mean* any of these things by these names, nor do they (in Mill's sense) form any part of the connotation of the names. Compare two such names as "John Duke Coleridge" and "the Lord Chief Justice of England." They denote the same individual, and I should recognise John Duke Coleridge, and the Lord Chief Justice of England by the same attributes; but the names are not equivalent,—the one is given as a mere mark of a certain individual to distinguish him from others, and it has no further signification; the other is given on account of the performance of certain functions, which ceasing the name would cease to apply. Surely there is a distinction here, and one which it is important that we should not overlook.

Nor is it true that such a name as "John Smith" connotes "Teuton, male, &c." John Smith might be a race-horse, or a negro, or the pseudonym of a woman, as in

CHAP. II.]   TERMS.   21

the case of George Eliot. In none of these cases could a
name be said to be misapplied as it would if a horse were
called a man, or a negro a Teuton, or a woman a
male.

But it may fairly be said that in a certain sense many
proper names do suggest something, that at any rate they
were chosen in the first instance for a special reason. For
example, Strongi'th'arm, Smith, Jungfrau. Such names
however even if in a certain sense connotative when first
imposed soon cease to be connotative in the way in which
other names are connotative. Their application is in no
way dependent on the continuance of the attribute with
reference to which they were originally given. As Mill puts
it, "*the name once given is independent of the reason.*" Thus,
a man may in his youth have been strong, but we should
not continue to call him strong when he is in his dotage;
whilst the name Strongi'th'arm once given would not be
taken from him. The name "Smith" may in the first
instance have been given because a man plied a certain
handicraft, but he would still be called by the same name
if he changed his trade, and his descendants continue to be
called Smiths whatever their occupations may be. Nor can
it be said that the name necessarily implies ancestors of
the same name.

Proper names of course become connotative when they
are used to designate a certain type of person; for example,
a Diogenes, a Thomas, a Don Quixote, a Paul Pry, a
Benedick, a Socrates. But, when so used, such names
have really ceased to be proper names at all; and they
have come to possess all the characters of general names.

15. Discuss the question whether the following
terms are respectively connotative or non-connota-

tive :—Westminster Abbey, the Mikado of Japan,
Barmouth.   [L.]

16. Enquire whether the following names are
respectively connotative or non-connotative:—Caesar,
Czar, Lord Beaconsfield, the highest mountain in
Europe, Mont Blanc, the Weisshorn, Greenland, the
Claimant, the pole star, Homer, a Daniel come to
judgment.

17. Can any abstract names possess both deno-
tation and connotation?

In Fowler's use of the term all abstract names are con-
notative, that is, they at once suggest or imply attributes ;
while none are denotative, that is, they do not denote
individuals or groups of individuals.   Professor Fowler
himself admits that it sounds paradoxical to say that
abstract names are not denotative, but he is of opinion that
the employment of the expressions in his sense would
simplify the statement and explanation of many logical
difficulties.   I am inclined to think that the present is a
case in point.

Mill holds that while most abstract names are non-con-
notative, still "even abstract names, though the names only
of attributes, may in some instances be justly considered as
connotative ; for attributes themselves may have attributes
ascribed to them ; and a word which denotes attributes may
connote an attribute of those attributes" (*Logic*, I. p. 33).   I
have some difficulty in interpreting this passage.   Suppose
that we have a connotative abstract name denoting the attri-
bute A and connoting the attribute B ; now a connotative
name is always defined by means of its connotation, and we
shall therefore define our term by saying that it connotes B

without any reference whatever to A. What then will distinguish it from the concrete term denoting whatever possesses B? The solution of the difficulty seems to be that when we talk of one attribute having another ascribed to it, the term denoting it becomes concrete rather than abstract. Comparing Mill's definitions of an abstract name and of a connotative name, I fail to understand how the same name can be both [1].

18. Explain and discuss the statement :—" In a series of common terms arranged in regular subordination to one another, the denotation and connotation vary inversely."

19. Explain the following statements :—

(*a*) If a term be abstract, its denotation is the same as the connotation of the corresponding concrete?

(*b*) Of the denotation and connotation of a term, one may, both cannot, be arbitrary.

(*c*) Names with indeterminate connotation are not to be confounded with names which have more than one connotation.

20. Verbal and Real Propositions.

A Verbal Proposition is one in which the connotation

---

[1] Mr Killick in his *Handbook of Mill's Logic* makes Mill include in the class of connotative names such abstract names as are the names of groups of attributes (*e.g., humanity*). I do not think that Mill himself intended this, nor do I think that the view is a correct one (*i.e.*, according to Mill's own usage of terms). If an abstract name has both denotation and connotation because it is the name of a group of attributes, on what principle shall we distinguish between the attributes that it denotes and those that it connotes?

of the predicate is a part or the whole of the connotation
of the subject. Bain describes the verbal proposition as
"the notion under the guise of the proposition"; and it is
certainly convenient to discuss verbal propositions in con-
nection with the connotation of names or the intension of
concepts. The most important class of verbal propositions
are definitions, the essential function of which is to analyse
the connotation of names[1]. The least important class are
absolutely tautologous or identical propositions, *e.g.*, all *A*
is *A*, a man is a man.

Real Propositions, on the other hand, "predicate of a
thing some fact not involved in the signification of the
name by which the proposition speaks of it; some attribute
not connoted by that name."

The same distinction is also expressed by the pairs of
terms, analytic[2] and synthetic, explicative[2] and ampliative,
essential[2] and accidental.

[1] Besides propositions giving such an analysis more or less com-
plete, the following classes of propositions are frequently included under
the head of verbal propositions: where the subject and predicate are
both proper names, *e.g.*, Tully is Cicero; where they are dictionary
synonyms, *e.g.*, wealth is riches, a story is a tale, charity is love.

All such propositions however can hardly be brought under the head
of verbal propositions as defined in the text. At any rate if we have
decided that a proper name is not connotative, it is clear that in no
proposition having a proper name for its subject can the predicate be
any part of the connotation of the subject.

To include these classes we must define a verbal proposition as a
proposition which is wholly concerned with the meaning or application
of names, a real proposition as one which is concerned with things or
qualities.

Even with these definitions, however, while it is a verbal proposition
to say that Tully is Cicero (*i.e.*, that these names have the same appli-
cation), it is a real proposition to say that Tully is an individual who is
also denoted by the name Cicero.

[2] It should be carefully observed that while the term *verbal* is some-

21. Which of the following propositions should you regard as *Real*, and why?

Homer wrote the Iliad,

Instinct is untaught ability,

Instinct is hereditary experience.   [C.]

"Homer wrote the Iliad" is regarded by Bain as a verbal predication. "We know nothing about Homer except the authorship of the Iliad. We have not a meaning to attach to the subject of the proposition, 'Homer', apart from the predicate, 'wrote the Iliad.' The affirmation is nothing more than that the author of the Iliad was called Homer" (*Logic, Deduction,* p. 67). Taking the definition of verbal proposition given in the text, and holding that no proper name is connotative, this view must clearly be rejected. If however by a verbal proposition we mean one that relates in any way to the application of names, (*i.e.,* taking the definition given in the note), there may be something to say for it. But is it true that we attach nothing more to "Homer" than "wrote the Iliad"? Do we not, for example, attach to "Homer" the authorship of other poems, and also an individuality [1]? If it is the fact that the Iliad was the work of various authors, as has been

times stretched so as to include such a proposition as "Tully is Cicero," this is never the case with the terms analytic, explicative, essential. These terms are strictly limited to propositions which give no information whatever (even with regard to the application of names) to any one who is fully acquainted with the connotation or intension of the subject term.

[1] I do not of course mean that this is the connotation of "Homer," for I hold that no proper names are connotative. I mean that *Homer* denotes for me a certain individual who was a Greek, who lived prior to a certain date, and who was the author of certain poems other than the Iliad.

asserted, would not the proposition become false? Still, we should perhaps admit that we have here a limiting case. Some light may be thrown on the point thus raised by an answer once sent in by an examinee: "The accepted opinion is that the Iliad was not written by Homer, but by another man of the same name."

"Instinct is untaught ability" and "Instinct is hereditary experience" may be regarded as verbal and real respectively.

22. Is it a verbal proposition to say that it is hotter in summer than in winter?

Examine the following statements: A free institution is a contradiction in terms; so is a perfect creature.   [v.]

23. If all x is y, and some x is z, and p is the name of those z's which are x; is it a verbal proposition to say that all p is y?   [v.]

24. Give one example of each of the following,— (i) a collective general name, (ii) a singular abstract name, (iii) a connotative abstract name, (iv) a connotative singular name; or, if you deny the possibility of any of these combinations, state clearly your reasons.

CHAPTER III.

25.   Positive and Negative Terms.

The essential distinction between positive and negative names as ordinarily understood may be expressed as follows :—a *positive* name implies the *presence* of certain definite attributes; a *negative* name implies the *absence* of one or other of certain definite attributes.

"Every name," as remarked by De Morgan, "applies to everything positively or negatively"; for example, everything either *is* or *is not* a horse.   Every name then divides all things in the universe into two classes.   Of one of these it is itself the name; and a corresponding name can be framed to denote the other.   This pair of names, which between them denote the whole universe, are respectively positive and negative.   But which is which?   Which is the negative name, since each positively denotes a certain class of objects?   The distinction lies in the manner in which the class is determined.   We may say that in a certain sense a strictly negative name has not an independent connotation of its own; its denotation is determined by the connotation of the corresponding positive name.   It denotes an indefinite and unknown class outside a definite and limited class.   In other words, by means of its connotation

we first mark off the class denoted by the positive name,
and then the negative name denotes what is left. The fact
that its denotation is thus determined is the distinctive
characteristic of the negative name.

We have here supposed that between them the positive
name and the corresponding negative name exhaust the
whole universe. But something different from this is often
meant by a negative name. Thus De Morgan considers
that *parallel* and *alien* are negative names. "In the
formation of language, a great many names are, as to their
original signification, of a purely negative character : thus,
parallels are only lines which do *not* meet, aliens are men
who are *not* Britons (*i.e.*, in our country)" (*Formal Logic*,
p. 37). But these names clearly have not the thorough-
going negative character that I have just been ascribing to
negative names. The difference will be found to consist in
this, that in the sense in which *alien* is a negative name,
the positive and negative names (Briton and alien) do not
between them exhaust the entire universe, but only a limited
universe, namely, in the given case, that constituted by the
inhabitants of Great Britain. We may perhaps distinguish
between names *absolutely negative*, where the reference is to
the entire universe ; and names *relatively negative*, where
the reference is only to some limited universe.

Now it will be seen that in the use of such a term as
not-white there is a possible ambiguity ; we must decide
whether in any given instance the name is to be regarded
as absolutely or only as relatively negative. Mill chooses
the former alternative ; "not-white," he says, "denotes all
things whatever except white things." De Morgan and
Bain however consider that in such a case the reference is
not to the whole universe but to some particular universe
only. Thus, in contrasting white and not-white we are

referring solely to the universe of colour; *not-white* does not include everything in nature except white things, but only things that are black, red, green, yellow, &c., that is, all *coloured* things except such as are white[1]. Whately and Jevons agree with Mill; and from a logical point of view I think they are right. Or rather I would say that two such terms as S and not-S must between them exhaust the *universe of discourse*, whatever that may be; and we must not be precluded from making this, if we care to do so, the entire universe of existence. That is, not-S *may be* called upon to assume the absolutely negative character[2]. For if we are unable to denote by not-S all things whatsoever except S, it is difficult to see in what way we shall be able to denote these when we have occasion to refer to them. On the other hand, we must also be empowered to indicate a limitation to a particular universe where that is intended. By not-S then referred to without qualification expressed or implied by the context I would understand the absolute negative of S; but I should be quite prepared to find a limitation to some more restricted universe in any particular instance.

It should be noted that in the case of a limited universe it is sometimes difficult to say which of the pair of contrasted names is really to be regarded as the negative name. For example, De Morgan says that *parallel* is a negative name, since parallel lines are simply lines that do not meet. But we might also define them as lines such that

[1] Thus, on Bain's view it would be incorrect to say that an immaterial entity such as honesty was not-white.

[2] On this view, "not-white" might be used to denote not merely coloured things that are not white, but also things that are not coloured at all. It would for example be correct to say that honesty was not-white.

if another line be drawn cutting them both, the alternate
angles are equal to one another; and then the name appears
as a positive name. Similarly in the universe of property,
as pointed out by De Morgan, *personal* and *real* are
respectively the negatives of each other; but if we are to
call one positive and the other negative, it is not quite clear
which should be which.

For a suggestion of Mr Monck's as to the definition of
negative terms, see section 29.

26. Privative Names.

To the distinction between positive and negative names,
Mill adds a class of names called *privative.* " A privative
name is equivalent in its signification to a positive and a
negative name taken together; being the name of some-
thing which has once had a particular attribute, or for
some other reason might have been expected to have it, but
which has it not. Such is the word *blind*, which is not
equivalent to *not seeing*, or to *not capable of seeing*, for it
would not, except by a poetical or rhetorical figure, be
applied to stocks and stones" (*Logic*, I. p. 44). Perhaps
also *idle*, which Mill gives as a negative, should rather be
regarded as a privative term. It does not mean merely
"not-working," but "not-working where there is the capacity
to work." We should hardly speak of a stone as being
"idle."

The distinction here indicated does not appear to be of
logical importance.

27. How far is it true that, as ordinarily under-
stood, negative terms have a definite connotation,
while in Logic they have not? So far as it is true,
how would you explain the fact?   [w.]

28. *Contradictory* and *contrary* terms.

A positive term and its corresponding negative term are
called *contradictories*.   A pair of contradictory terms are so
related that between them they exhaust the entire universe
to which reference is made, whilst in that universe there is
no individual of which both can be at the same time
affirmed.   The nature of this relation is expressed in the
two laws of Contradiction and Excluded Middle.   Nothing
is at the same time both X and not-X; Everything is X
or not-X.   For the application of the above to complex
terms, see Part IV.

The *contrary* of a term is usually defined as the term
denoting that which is furthest removed from it in some
particular universe ; *e.g.*, black and white, wise and foolish.
Two contraries may in some cases happen to make up
between them the whole of the universe in question, *e.g.*,
Briton and alien ; but this is not necessary, *e.g.*, black and
white.   It follows that although two contraries cannot both
be true of the same thing at the same time, they may both
be false.

The above may be called the *material* contrary.   In the
case of complex terms, we may also assign a formal con-
trary, as is shewn in Part IV.

29.   Illustrate Mill's statement that "names which
are positive in form are often negative in reality, and
others are really positive though their form is nega-
tive."

The fact that a really positive term is sometimes negative
in form results from the circumstance that the negative pre-
fix is sometimes given to the contrary of a term.   But we
have seen that a term and its contrary may both be positive.

For example, pleasant and unpleasant; "the word *un-pleasant*, notwithstanding its negative form, does not connote the mere absence of pleasantness, but a less degree of what is signified by the word *painful*, which, it is hardly necessary to say, is positive." On the other hand, some names positive in form may be regarded as relatively negative, *e.g.*, parallel, alien. I do not however think that an absolutely negative name can be found that is positive in form.

But for purposes of formal logic it does not much concern us whether any given term is positive or negative. What the formal logician is really concerned with is the relation between contradictory terms. Not-*S* is the contradictory of *S*, and *S* is the contradictory of not-*S*, whichever of the terms may be more strictly the positive and the negative respectively.

Mr Monck, in his valuable *Introduction to Logic*, p. 104, suggests that it might be "better to define a Negative term as a term negative in form, (*i.e.*, a term in which 'non,' 'un,' 'in,' 'mis,' or some other negative particle occurs)." In my opinion, this suggestion might without disadvantage be adopted.

30. Truth applies, it is said, only to propositions. If, then, a simple term is not capable of truth, it must be false; because everything must be either true or false. Solve this difficulty. [L.]

31. "For every positive concrete name a corresponding negative one might be framed." Illustrate the meaning of this statement, and find the precise negatives of the positive terms *Man*, *Physician*, *Red*, *Thing*. [L.]

32. Relative Names.

"A name is relative, when, over and above the object which it denotes, it implies in its signification the existence of another object, also deriving a denomination from the same fact which is the ground of the first name." (Mill, *Logic,* I. p. 47.)

Jevons considers that all terms are in one sense relative. By the law of relativity, consciousness is possible only under circumstances of change. Every term therefore implies its negative as an object of thought. For example, take the term *man.* It is an ambiguous term, and in many of its meanings it is strikingly relative,—for example, as opposed to master, to officer, to woman, to wife, to boy. If in any sense it is *absolute, i.e.,* not relative, it is when opposed to not-man; but even in this case it may be said to be relative to not-man. To avoid this difficulty, Jevons remarks, "Logicians have been content to consider as relative terms those only which imply some peculiar and striking kind of relation arising from position in time or space, from connexion of cause and effect, &c.; and it is in this special sense therefore that the student must use the distinction."

It is a little doubtful however whether every name can be said to imply its negative *in its signification.* Because all *things* are relative does it necessarily follow that all *terms* are relative? The matter is of no great importance, and at any rate the difficulty might be avoided by defining a relative term as one which implies in its signification the existence of another object, *other than its mere negation.*

The fact or facts constituting the ground of both correlative names is called the *fundamentum relationis.* For example, in the case of partner, the fact of partnership; in the case of husband and wife, the facts which constitute

the marriage tie; in the case of shepherd and sheep, the acts of tending and watching which the former exercises over the latter.

Sometimes the relation which each correlative bears to the other is the same; for example, in the case of partner, where the correlative name is the same name over again. Sometimes it is not the same; for example, father and son, husband and wife.

33. Describe in logical phrase the character of the following words:—man, Peter, humanity, the sun, post, idle, unpleasant, daughter. [C.]

In dealing with any term for logical purposes, we must first of all determine whether it is *univocal*, that is, used in one definite sense only, or *equivocal* (or ambiguous), that is, used in more senses than one. In the latter case, we may find that its logical characteristics vary according to the sense in which it is used.

34. What are the logical characteristics of the terms:—beauty, immortal, slave, England, Paradise, friendship, law, sovereign, the Times, the Arabian Nights, George Eliot, Mrs Grundy, Vanity Fair, sleep, truth, selfish, ungenerous, nobility, treason?

PART II.

PROPOSITIONS.

CHAPTER I.

35. Categorical, Hypothetical and Disjunctive Propositions.

For logical purposes, a *Proposition* may be defined as "a sentence indicative or assertory," (as distinguished, for example, from sentences imperative or exclamatory); in other words, a proposition is a sentence making an affirmation or denial, as—All *S* is *P*, No vicious man is happy.

A proposition is *Categorical* if the affirmation or denial is absolute, as in the above examples. It is *Hypothetical* if made under a condition, as—If *A* is *B*, *C* is *D*; Where ignorance is bliss, 'tis folly to be wise. It is *Disjunctive* if made with an alternative, as—Either *P* is *Q*, or *X* is *Y*; He is either a knave or a fool[1].

[1] It should be observed that in a disjunctive proposition there may be two distinct subjects as in the first of the above examples, or only one as in the second. Disjunctive propositions in which there is only one distinct subject are the more amenable to logical treatment.

[The above threefold division is adopted by Mansel. It is perhaps more usual to commence with a twofold division, the second member of which is again subdivided, the term Hypothetical being employed sometimes in a wider and sometimes in a narrower sense.   To prevent confusion, it may be helpful to give the following table of the usage of one or two modern logicians with regard to this division.

Whately, Mill and Bain :—

> 1.   Categorical.
> 2.   Hypothetical, $\left\{\begin{array}{l}(1)\ \text{Conditional.}\\(2)\ \text{Disjunctive.}\end{array}\right.$
>   or Compound,
>   or Complex.

Hamilton and Thomson :—

> 1.   Categorical.
> 2.   Conditional. $\left\{\begin{array}{l}(1)\ \text{Hypothetical.}\\(2)\ \text{Disjunctive.}\end{array}\right.$

Fowler (following Boethius) :—

> 1.   Categorical.
> 2.   Conditional $\left\{\begin{array}{l}(1)\ \text{Conjunctive.}\\(2)\ \text{Disjunctive.}\end{array}\right.$
>   or Hypothetical.

Mansel, as I have already remarked, gives at once a threefold division.

> 1.   Categorical.
> 2.   Hypothetical or Conditional.
> 3.   Disjunctive.

He states his reasons for his own choice of terms as follows :—" Nothing can be more clumsy than the employment of the word *conditional* in a specific sense, while its Greek equivalent, *hypothetical*, is used generically. In Boethius, both terms are properly used as synonymous, and generic ; the two species being called *conjunctivi, conjuncti,*

or *connexi*, and *disjunctivi* or *disjuncti*. With reference to modern usage, however, it will be better to contract the Greek word than to extend the Latin one. *Hypothetical* in the following notes, will be used as synonymous with *conditional*" (Mansel's edition of *Aldrich*, p. 103).]

36. A logical analysis of the Categorical Proposition.

In logical analysis, the categorical proposition always consists of three parts, namely, two terms which are united by means of a copula.

The *subject* is that term about which affirmation or denial is made; it represents some notion already partially determined in our mind, and which it is our aim further to determine.

The *predicate* is that term which is affirmed or denied of the subject; it enables us further to determine the subject, *i.e.*, to enlarge our knowledge with regard to it.

The *copula* is the link of connection between the subject and the predicate, and consists of the words *is* or *is not* according as we affirm or deny the latter of the former.

In attempting to apply the above analysis to such a proposition as "All that love virtue love angling," we find that, as it stands, the copula is not separately expressed. It may however be written,—

subj.   cop.   pred.
All lovers of virtue | are | lovers of angling;

and in this form the three different elements of the logical proposition are made distinct. This analysis should always be performed in the case of any proposition that may at first present itself in an abnormal form. A difficulty that may sometimes arise in discriminating the subject

and the predicate is dealt with subsequently,—compare section 50.

The older logicians distinguished propositions *secundi adjacentis*, and propositions *tertii adjacentis*. In the former, the copula and the predicate are not separated ; *e.g.*, The man runs, All that love virtue love angling. In the latter, the copula and the predicate are made distinct ; *e.g.*, The man is running, All lovers of virtue are lovers of angling. A categorical proposition, therefore, when expressed in exact logical form, is *tertii adjacentis.*

37. *Exponible, copulative, exclusive, exceptive* propositions.

Propositions that are resolvable into more propositions than one have been called *exponible*, in consequence of their susceptibility of analysis. *Copulative* propositions are formed by a direct combination of simple propositions, *e.g.*, P is both Q and R (*i.e.*, P is Q, P is R), A is neither B nor C (*i.e.*, A is not B, A is not C); they form one class of exponibles. *Exclusive* propositions contain some such word as "only," thereby limiting the predicate to the subject ; *e.g.*, Only S is P. This may be resolved into S is P, and P is S. Propositions of this kind also are therefore exponibles. *Exceptive* propositions limit the subject by such a word as "unless" or "except"; *e.g.*, A is X, unless it happens to be B. These too may perhaps be regarded as exponible propositions.

38. The Quantity and Quality of Propositions.

The *Quality* of a proposition is determined by the copula, being *affirmative* or *negative* according as the copula is of the form "is" or "is not."

Propositions are also divided into *universal* and *parti-*

cular, according as the affirmation or denial is made of the whole or only of a part of the subject. This division of Propositions is said to be according to their *Quantity*.

Combining the two principles of division, we get four fundamental forms of propositions :—

(1) the *universal affirmative*, All S is P, usually denoted by the symbol **A** ;

(2) the *particular affirmative*, Some S is P, usually denoted by the symbol **I** ;

(3) the *universal negative*, No S is P, usually denoted by the symbol **E** ;

(4) the *particular negative*, Some S is not P, usually denoted by the symbol **O**.

These symbols A, I and E, O are taken from the Latin words *affirmo* and *nego*, the affirmative symbols being the first two vowels of the former, and the negative symbols the two vowels of the latter.

Besides these symbols, it will also be found convenient sometimes to use the following,—

$$SaP = \text{All } S \text{ is } P;$$
$$SiP = \text{Some } S \text{ is } P;$$
$$SeP = \text{No } S \text{ is } P;$$
$$SoP = \text{Some } S \text{ is not } P.$$

The above are useful when we wish that the symbol which is used to denote the proposition as a whole should also indicate what symbols have been chosen for the subject and the predicate respectively. Thus,

$$MaP = \text{All } M \text{ is } P;$$
$$PoQ = \text{Some } P \text{ is not } Q.$$

The universal negative should be written in the form *No S is P*, not *All S is not P*; for the latter would usually

be understood to be merely particular. Thus, All that glitters is not gold is really an **O** proposition, and is equivalent to—Some things that glitter | are not | gold.

39. Indefinite Propositions.

According to Quantity, Propositions have sometimes been divided into (1) Universal, (2) Particular, (3) Singular, (4) Indefinite. Singular propositions are discussed in the following section.

By an *Indefinite* Proposition is meant one "in which the Quantity is not explicitly declared by one of the designatory terms *all, every, some, many*, &c." We may perhaps say with Hamilton that *indesignate* or *preindesignate* would be a better term to employ. There can be no doubt that, as Mansel remarks, "The true indefinite proposition is in fact the particular; the statement 'some *A* is *B*' being applicable to an uncertain number of instances, from the whole class down to any portion of it. For this reason particular propositions were called indefinite by Theophrastus" (*Aldrich*, p. 49).

Some indesignate propositions are no doubt intended to be understood as universals, *e.g.*, Comets are subject to the law of gravitation ; but in such cases before we deal with the proposition logically it is better that the word *all* should be explicitly prefixed to it. If we are really in doubt with regard to the quantity of the proposition it must logically be regarded as particular.

Other designations of quantity besides *all* and *some*, *e.g.*, *most*, are discussed in section 41.

The term *indefinite* has also been applied to propositions in another sense. According to Quality, instead of the twofold division given in the preceding example, a threefold division is sometimes adopted, namely into affirmative,

negative, and infinite or *indefinite.*   For further explanation, see section 44.

40.   Singular Propositions.

By a *Singular* or *Individual* Proposition is meant a proposition of which the subject is a singular term, one therefore in which the affirmation or denial is made but of a single specified individual; *e.g.,* Brutus is an honourable man; Much Ado about Nothing is a play of Shakespeare's; My boat is on the shore.

Singular propositions may usually be regarded as forming a sub-class of Universal propositions, since in every singular proposition the affirmation or denial is of the *whole* of the subject.   Such propositions have however certain peculiarities of their own, as we shall note subsequently; *e.g.,* they have not like other universal propositions a contrary distinct from their contradictory.

Hamilton distinguishes between Universal and Singular Propositions, the predication being in the former case of a *Whole Undivided,* and in the latter case of a *Unit Indivisible.* This separation is sometimes useful; but I think it better not to make it absolute. A singular proposition may without risk of confusion be denoted by one of the symbols A or E; and in syllogistic inferences, a singular may always be regarded as equivalent to a universal proposition.   The use of independent symbols for affirmative and negative singular propositions would introduce considerable additional complexity into the treatment of the Syllogism; and for this reason alone it seems desirable as a rule to include particulars under universals.   We may however divide universal propositions into *General* and *Singular,* and we shall then have terms whereby to call attention to the distinction wherever it may be necessary or useful to do so.

There is a certain class of propositions with regard to which there is some difference of opinion as to whether they should be regarded as singular or particular; for example, such as the following: A certain man had two sons; A great statesman was present. Mansel (*Aldrich*, p. 49) decides that they should be dealt with as particulars, and I think rightly, on the ground that if we have two such propositions, "a certain man" or "a great statesman" being the subject of each, we cannot be sure that the same individual is referred to in both cases. Sometimes however the context may enable us to decide the case differently.

There are propositions of another kind with a singular term for subject about which a few words may be said; namely, such propositions as—Browning is sometimes obscure; That boy is sometimes first in his class. These propositions may be treated as universal with a somewhat complex predicate, (and it should be noted that in bringing propositions into logical form we are frequently compelled to use very complex predicates); thus :—

Browning | is | a poet who is sometimes obscure.

That boy | is | a boy who is sometimes first in his class.

By a certain transformation however these propositions may also be dealt with as particulars, and such transformation may sometimes be convenient; thus, Some of Browning's writings are obscure, Some of the boy's places in his class are the first places. But when the proposition is thus modified, the subject is no longer a singular term.

41. The logical signification of the words *some, most, few, all, any.*

Some may mean merely "some at least," *i.e.*, not none, or it may carry the further implication, "some at most," *i.e.*, not all. Professor Bain is probably right in saying (*Logic*,

Deduction, p. 81) that in ordinary speech the latter meaning is the more usual. With most modern logicians, however, the logical implication of some is limited to some at least, not exclusive of all. Using the word in this sense, if we want to express "some, but not all, *S* is *P*," we must make use of two propositions,

> Some *S* is *P*,
> Some *S* is not *P*.

The particular then is not exclusive of the universal. As already suggested, it is indefinite, though with a certain limit; that is, it is indefinite so far that it may apply to any number from a single one up to all, but on the other hand it is definite so far as it excludes "none."

It may be added that in regarding "some" as implying no more than *at least one*, we are probably again departing from the ordinary usage of language, which would regard it as implying *at least two*.

[It should perhaps be noted that on rare occasions "some" may have a slightly different implication. For example, the proposition "Some truth is better kept to oneself" may be so emphasized as to make it perfectly clear to what particular kind of truth reference is made. This is however extra-logical. Logically the proposition must be treated as particular, or it must be written in another form, "All truth of a certain specified kind is better kept to oneself." Thus, Spalding remarks (*Logic*, p. 63), "The logical 'some' is totally indeterminate in its reference to the constitutive objects. It is always *aliqui*, never *quidam*; it designates some objects or other of the class, not some certain objects definitely pointed out."]

Most is to be interpreted "at least one more than half." *Few* has a negative force, "Few *S* is *P*" being equivalent

to "Most S is not P"; (with perhaps the further implication "although *some* S is P"; thus Few S is P is given by Kant as an example of the *exponible* proposition, on the ground that it contains both an affirmation and a negation, though one of them in a concealed way). Formal logicians (excepting De Morgan and Hamilton) have not as a rule recognized these additional signs of quantity; and it is true that in many logical combinations we are unable to regard them as more than particular propositions, Most S is P being reduced to Some S is P, and Few S is P to Some S is not P. Sometimes however we are able to make use of the extra knowledge given us; *e.g.*, from Most M is P, Most M is S we can infer Some S is P, although from Some M is P, Some M is S we can infer nothing.

It should be observed that *A few* has not the same signification as *Few*, but must be regarded as affirmative, and, generally, as simply equivalent to *some; e.g.*, A few S is P = Some S is P. Sometimes, however, it means "a small number," and in this case the proposition is perhaps best regarded as singular, the subject being collective. Thus " a few peasants successfully defended the citadel " may be rendered " a small band of peasants successfully defended the citadel," rather than "some peasants successfully defended the citadel," since the stress is intended to be laid at least as much on the paucity of their numbers as on the fact that they were peasants. In this case, the proposition would be **A**, not **I**.

It may here be remarked that in all cases, where we are dealing with propositions which as originally stated are not in a logical form, the first problem in reducing them to logical form is one of interpretation, and we must not be surprised to find that in many cases different methods of interpretation lead to different results. No confusion will

ensue if we make it perfectly clear what we do regard as the logical form of the proposition, and also how we have arrived at our result.

All is ambiguous, so far as it may be used either distributively or collectively. In the proposition "All the angles of a triangle are less than two right angles" it is used distributively, the predicate applying to each and every angle of a triangle taken separately. In the proposition " All the angles of a triangle are equal to two right angles " it is used collectively, the predicate applying to all the angles taken together, and not to each separately.

Any as the sign of quantity of the subject of a categorical proposition, (*e.g.*, any *S* is *P*), is logically equivalent to "all" in its distributive sense. Whatever is true of any member of a class taken at random is necessarily true of the whole of that class. When not the subject of a categorical proposition, *any* may have a different signification. For example, in the hypothetical proposition, If any *A* is *B*, *C* is *D*, it has the same indefinite character which we logically ascribe to "some"; since the antecedent condition is satisfied if a single *A* is *B*. The proposition might indeed be written—If one or more *A* is *B*, *C* is *D*.

42. Examine the logical signification of the italicised words in the following propositions :—

Some are born great.
Few are chosen.
All is not lost.
All men are created equal.
All that a man hath will he give for his life.
If *some A* is *B*, *some C* is *D*.
If *any A* is *B*, *any C* is *D*.
If *all A* is *B*, *all C* is *D*.

43. Distinguish the collective and distributive use of the word *all* in the following propositions :

(1) All Albinos are pink-eyed people ;

(2) Omnes apostoli sunt duodecim ;

(3) Non omnis moriar ;

(4) Non omnia possumus omnes ;

(5) All men find their own in all men's good,
    And all men join in noble brotherhood.

(6) Not all the gallant efforts of the officers and escort of the British Embassy at Cabul were able to save them.

[Jevons, *Elementary Lessons in Logic*, p. 297.   **Studies in Deductive Logic**, pp. 19, 28.]

44. *Infinite* or *indefinite* terms and propositions.

Infinite and *indefinite* are designations applied to terms having a thoroughgoing negative character ; to such a term for example as " not-white," understood as denoting not merely coloured things other than white, but the whole infinite or indefinite class of things of which " white " cannot be truly affirmed, including such entities as Mill's *Logic*, a dream, Time, a soliloquy, New Guinea, the Seven Ages of Man.

It is however to be observed that if symbols are used, it is impossible to say which of the terms S or not-S really partakes of this indefinite character, since, for example, there is nothing to prevent our having originally written S for " not-white," in which case " white " becomes not-S, and S is the really *indefinite* or *infinite* term.

Following out the above idea, propositions were divided by Kant into three classes in respect of Quality, namely, affirmative—A is B, negative—A is not B, and *infinite* (or

indefinite)—*A* is not-*B*. Logically however the last proposition (which is equivalent to the second in meaning) must be regarded as simply affirmative. As just shewn, it is impossible to say which of the terms *B* or not-*B* is really infinite or indefinite ; and it is therefore also impossible to say which of the propositions "*A* is *B*" or "*A* is not-*B*" is really infinite or indefinite. Logically then they must be regarded as belonging to the same type of proposition, and we have to fall back upon the two-fold division into affirmative and negative[1].

45. Can distinctions of Quality and Quantity be applied to Hypothetical and Disjunctive Propositions?

The parts of the *Hypothetical* Proposition are called the Antecedent and the Consequent. Thus, in the proposition, "If *A* is *B*, *C* is *D*," the Antecedent is "*A* is *B*," the Consequent is "*C* is *D*". The Quality of the Hypothetical Proposition depends upon the Quality of the Consequent. Thus, the proposition If *A* is *B*, *C* is not *D*, is to be considered negative. Hypothetical propositions may also be regarded as Universal or Particular, according as the consequent is affirmed to follow from the antecedent in all or only in some cases. We have then the four fundamental types of proposition :—

(1) If *A* is *B*, *C* is *D*.   · **A.**
(2) In some cases in which *A* is *B*, *C* is *D*.   **I.**
(3) If *A* is *B*, *C* is not *D*.   **E.**
(4) In some cases in which *A* is *B*, *C* is not *D*.   **O.**

[1] It should be observed that, if we admit its use as above, the term *indefinite* as applied to propositions is ambiguous, since by an *indefinite* proposition we mean here something entirely different from what was called an indefinite proposition in section 39. In the one case the reference is to the Quality of the proposition, in the other case to its Quantity.

The student must be warned against treating such a proposition as "If any *A* is *B*, some *C* is *D*" as particular[1]. Regarded separately the antecedent and the consequent in this example are both particular; but the connection between them is affirmed universally, the proposition asserting that "*in all cases* in which any *A* is *B*, some *C* is *D*."

It should be observed that in a considerable number of cases, the hypothetical is of the nature of a singular proposition, the event referred to in the antecedent being in the nature of things one which can happen but once; *e.g.*, If I perish in the attempt, I shall not die unavenged.

To the *Disjunctive* Proposition we are unable to apply distinctions of Quality. The proposition, Neither *P* is *Q* nor *X* is *Y* states no alternative, and is therefore not disjunctive at all. Distinctions of Quantity are however still applicable. Thus,

Universal,—Either *P* is *Q* or *X* is *Y*.

Particular,—In some cases either *P* is *Q* or *X* is *Y*.

It is again to be observed that frequently the disjunctive proposition is of the nature of a singular proposition, the reference being but to a single occasion on which it is asserted that one of the alternatives will hold good.

46. Determine the Quantity and Quality of the following propositions, stating precisely what you regard as the subject and predicate, or in the case

---

[1] I cannot agree with Hamilton (*Logic*, i. p. 248), in regarding the following as a particular hypothetical—If some Dodo is, then some animal is. The proposition is a little hard to interpret, but it seems to mean that if there is such a thing as a Dodo, then there is such a thing as an animal; and we must consider that a universal connection is here affirmed.

of hypothetical propositions, the antecedent and consequent of each :—

(1)   All men think all men mortal but themselves.

(2)   Not to know me argues thyself unknown.

(3)   To bear is to conquer our fate.

(4)   Berkeley, a great philosopher, denied the existence of Matter.

(5)   A great philosopher has denied the existence of Matter.

(6)   The virtuous alone are happy.

(7)   None but Irish were in the artillery.

(8)   Not every tale we hear is to be believed.

(9)   Great is Diana of the Ephesians !

(10)   All sentences are not propositions.

(11)   Where there's a will there's a way.

(12)   Some men are always in the wrong.

(13)   Facts are stubborn things.

(14)   He that increaseth knowledge increaseth sorrow.

(15)   None think the great unhappy, but the great.

(16)   He can't be wrong, whose life is in the right.

(17)   Nothing is expedient which is unjust.

(18)   Mercy but murders, pardoning those that kill.

(19)   If virtue is involuntary, so is vice.

(20)   Who spareth the rod, hateth his child.

47.   Analyse the following propositions, *i.e.*, express them in one or more of the strict categorical forms admitted in Logic :—

(i) No one can be rich and happy unless he is also temperate and prudent, and not always then.

(ii) No child ever fails to be troublesome if ill taught and spoilt.

(iii) It would be equally false to assert that the rich alone are happy, or that they alone are not. [v.]

(i) contains *two* statements which may be reduced to the following forms,—

All who are rich and happy | are | temperate and prudent.   **A.**

Some who are temperate and prudent | are not | rich and happy.   **O.**

(ii) may be written, All ill-taught and spoilt children are troublesome.   **A.**

(iii) Here two statements are given *false*, namely, the rich alone are happy; the rich alone are not happy.

We may reduce these false statements to the following,— all who are happy are rich; all who are not happy are rich. And this gives us these true statements,—

Some who are happy are not rich.   **O.**

Some who are not happy are not rich.   **O.**

The original proposition is expressed therefore by means of these two particular negative propositions.

48. The Distribution of Terms in a Proposition.

A term is said to be distributed when reference is made to *all* the individuals denoted by it; it is said to be undistributed when they are only referred to *partially*, *i.e.*, information is given with regard to a portion of the class denoted by the term, but we are left in ignorance with regard to the remainder of the class. It follows immediately

from this definition that the subject is distributed in a universal, and undistributed in a particular, proposition. It can further be shewn that the predicate is distributed in a negative, and undistributed in an affirmative proposition. Thus, if I say, All *S* is *P*, I imply that at any rate *some P* is *S*, but I make no implication with regard to the whole of *P*. I leave it an open question as to whether there is or is not any *P* outside the class *S*. Similarly if I say, Some *S* is *P*. But if I say, No *S* is *P*, in excluding the whole of *S* from *P*, I am also excluding the whole of *P* from *S*, and therefore *P* as well as *S* is distributed. Again, if I say, Some *S* is not *P*, although I make an assertion with regard to a part only of *S*, I exclude this part from the whole of *P*, and therefore the whole of *P* from it. In this case, then, the predicate is distributed, although the subject is not.

Summing up our results we find that

A distributes its subject only,

I distributes neither its subject nor its predicate,

E distributes both its subject and its predicate,

O distributes its predicate only.

49. How does the Quality of a Proposition affect its Quantity? Is the relation a necessary one? [L.]

By the Quantity of a Proposition must here be meant the Quantity of its Predicate, and we have shewn in the preceding section that this is determined by its Quality. The predicate is distributed in negative, undistributed in affirmative, propositions.

The latter part of the above question refers to Hamilton's doctrine of the Quantification of the Predicate. According to this doctrine, the predicate of an affirmative proposition is sometimes expressly distributed, while the predicate of a

negative proposition is sometimes given undistributed. For example, the following forms are introduced :—

> Some *S* is all *P*,
> No *S* is some *P*.

This doctrine is discussed and illustrated in Part III. chapter 9.

50. In doubtful cases how should you decide which is the subject and which the predicate of a proposition ? [V.]

The nature of the distinction between the subject and the predicate of a proposition may be expressed by saying that the subject is that of which something is affirmed or denied, the predicate is that which is affirmed or denied of the subject; or perhaps still better, the subject is that which we think of as the determined or qualified notion, the predicate that which we think of as the determining or qualifying notion.

Now, can we say that the subject always precedes the copula, and that the predicate always follows it ? In other words, can we consider the order of the terms to suffice as a criterion ? If the proposition is reduced to an equation, as in the doctrine of the quantification of the predicate, I do not see what other criterion we can take ; or we might rather say that in this case the distinction between subject and predicate itself fails to hold good. The two are placed on an equality, and we have nothing left by which to distinguish them except the order in which they are stated. This view is indicated by Professor Baynes in his *Essay on the New Analytic of Logical Forms.* In such a proposition, for example, as " Great is Diana of the

Ephesians," he would call " great " the subject, reading the proposition, however, " (Some) great is (all) Diana of the Ephesians."

But leaving this view on one side, we cannot say that the order of terms is always a sufficient criterion. In the proposition just quoted, " Diana of the Ephesians " would generally be accepted as the subject. What further criterion then can be given? In the case of **E** and **I** propositions, (propositions, as will be shewn, which can be simply converted), we must appeal to the context or to the question to which the proposition is an answer. If one term clearly conveys information regarding the other term, it is the predicate. It is also more usual that the subject should be read in extension and the predicate in intension. If none of these considerations are decisive, then I should admit that the order of the terms must suffice. In the case of **A** and **O** propositions, (propositions, as will be shewn, which cannot be simply converted), a further criterion may be added. From the rules relating to the distribution of terms in a proposition it follows that in affirmative propositions the distributed term, (if either term is distributed), is the subject; whilst in negative propositions, if only one term is distributed, it is the predicate. I am not sure that the inversion of terms ever occurs in the case of an **O** proposition; but in **A** propositions it is not infrequent. Applying the above to such a proposition as " Workers of miracles were the apostles," it is clear that the latter term is distributed while the former is not. The latter term is therefore the subject. A corollary from the rule is that in an affirmative proposition if one and only one term is singular that is the subject, since a singular is equivalent to a distributed term. This decides such a case as " Great is Diana of the Ephesians."

51. What do you consider to be respectively the subject and the predicate of the following sentences, and why?

(1) Few men attain celebrity.

(2) Blessed are the peacemakers.

(3) It is mostly the boastful who fail.

(4) Clematis is Traveller's Joy.   [v.]

52. What do you consider to be the essential distinction between the Subject and Predicate of a proposition? Apply your answer to the following :—

(1) From thence thy warrant is thy sword.

(2) That is exactly what I wanted.   [v.]

CHAPTER II.

53. The Opposition of Categorical Propositions.

Two propositions are said to be *opposed* to each other when they have the same subject and predicate respectively, but differ in quantity or quality or both[1].

Taking the propositions *SaP*, *SiP*, *SeP*, *SoP*, in pairs we find that there are four possible kinds of relation between them.

(1) The pair of propositions may be such that they cannot both be true, and they cannot both be false. This is called *contradictory* opposition, and subsists between *SaP* and *SoP*, and between *SeP* and *SiP*.

---

[1] This definition is given by Aldrich (p. 53 in Mansel's edition). Ueberweg however defines Opposition in such a way as to include only contradiction and contrariety (translation by Lindsay, p. 328); and Mansel remarks that "Subalterns are improperly classed as *opposed* propositions" (*Aldrich*, p. 59). Professor Fowler follows Aldrich's definition (*Deductive Logic*, p. 74), and I think wisely. We want some term to signify this general relation between propositions; and though it might be possible to find a more convenient term, I do not think that any confusion is likely to result from the use of the term *opposition* if the student is careful to notice that it is here used in a technical sense.

(2) They may be such that they cannot both be true, but they may both be false. This is called *contrary* opposition. *SaP* and *SeP*.

(3) They may be such that they cannot both be false, but they may both be true. *Subcontrary* opposition. *SiP* and *SoP*.

(4) From a given universal proposition, the truth of the particular having the same quality follows, but not *vice versa*. This is *subaltern opposition*, the universal being called the *subalternant*, and the particular the *subalternate* or the *subaltern*. *SaP* and *SiP*. *SeP* and *SoP*.

All these relations are indicated clearly in the ancient square of opposition.
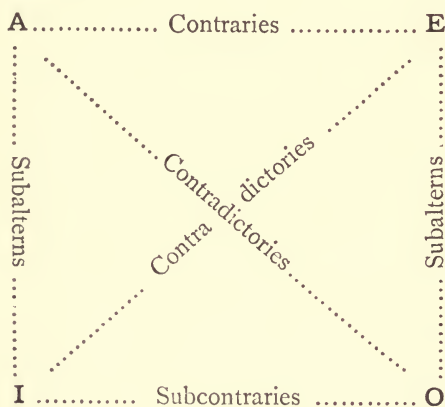
Propositions must of course be brought to such a form that they have the same subject and the same predicate before we can apply the terms of opposition to them; for example, All *S* is *P* and Some *P* is not *S* are not contradictories.

54. On the common view of the opposition of propositions what are the inferences to be drawn (1) from the truth, (2) from the falsity, of each of the four categorical propositions? [L.]

55. Explain the nature of the opposition between each pair of the following propositions :

None but Liberals voted against the motion.

Amongst those who voted against the motion were some Liberals.

It is untrue that those who voted against the motion were all Liberals.

56. Give the contradictory and the contrary of the following propositions :—

(1) A stitch in time saves nine.

(2) None but the brave deserve the fair.

(3) He can't be wrong whose life is in the right.

(4) The virtuous alone are happy.

(1) A stitch in time saves nine. This is to be regarded as a universal affirmative proposition, and we therefore have

Contradictory, Some stitches in time do not save nine. **I.**

Contrary, No stitch in time saves nine. **E.**

(2) None but the brave deserve the fair, = None who are not brave deserve the fair. **E.**

Contradictory, Some who are not brave deserve the fair. **I.**

Contrary, All who are not brave deserve the fair. **A.**

(3) He can't be wrong whose life is in the right.   **E.**

Contradictory, Some may be wrong whose lives are in the right. **I.**

Contrary, All are wrong whose lives are in the right. **A.**

(4) The virtuous alone are happy, = No one who is not virtuous is happy. **E.**

Contradictory, Some who are not virtuous are happy. **I.**

Contrary, All who are not virtuous are happy. **A.**

57. Give the contrary, contradictory, and subaltern of the following propositions :—

(1) All B.A.'s of the University of London have passed three examinations.

(2) All men are sometimes thoughtless.

(3) Uneasy lies the head that wears a crown.

(4) The whole is greater than any of its parts.

(5) None but solid bodies are crystals.

(6) He who has been bitten by a serpent is afraid of a rope.

(7) He who tries to say that which has never been said before him will probably say that which will never be repeated after him.

[Jevons, *Studies in Deductive Logic,* p. 58.]

58. Explain the technical terms " contrary " and " contradictory," applying them to the following propositions :—

(1) Few *S* are *P*.

(2) At any rate, he was not the only one who cheated.

(3) Two-thirds of the army are abroad.   [v.]

It is the same thing to deny the truth of a proposition and to affirm the truth of its *contradictory ;* and *vice versa.* The criterion of contradictory opposition is that *of the two propositions, one must be true and the other must be false ;* they cannot be true together, but on the other hand no mean is possible between them. The relation between two contradictories is mutual ; it does not matter which is given true or false, we know that the other is false or true accordingly. Every proposition has its contradictory, which may however be more or less complex in form.

The *contrary* of a given proposition goes beyond mere denial, and sets up a further assertion as far as possible removed from the original assertion. It declares not merely the falsity of the original proposition taken as a whole, but the falsity of every part of it.

It follows that if we cannot go beyond the simple denial of the truth of a proposition, then it has no contrary distinct from its contradictory. For example, in order simply to deny the truth of "some S is P," it is necessary to affirm that "no S is P," and it is impossible to go further than this in opposition to the given proposition. "Some S is P" has therefore no contrary as distinguished from its contradictory.

We may now apply the terms in question to the given propositions :—

(1) "Few S are P" = "Most S are not P," and we might hastily be inclined to say that the contradictory is "Most S are P." Both these propositions would however be false in the case in which exactly one half S was P. The true contradictory therefore is "At least one half S is P." The contrary is "All S is P." Similarly the contradictory of "Most S are P" is "At least one half S is not P"; and its contrary "No S is P."

These examples shew that if we once travel outside the

limits set by the old logic, and recognise the signs of quantity *most* and *few* as well as *all* and *some*, we soon become involved in numerical statements. Propositions of the above kind are therefore usually relegated to what has been called numerical logic, a topic discussed at length by De Morgan and to some extent by Jevons.

(2) "At any rate, he was not the only one who cheated." A question of interpretation is naturally raised here ; does the statement assert that *he* cheated, or is this left an open question? We may I think choose the latter alternative. What the speaker intends to lay stress upon is that some others cheated at any rate, whatever may have been the case with him. The contradictory then becomes "No others cheated"; and we have no distinct contrary.

(3) "Two-thirds of the army are abroad." This may mean "At least two-thirds of the army are abroad," or "Exactly two-thirds of the army are abroad."

On the first interpretation, the contradictory is "Less than two-thirds of the army are abroad"; and the contrary "None of the army are abroad."

On the second interpretation, the contradictory is "Not exactly two-thirds of the army are abroad," *i.e.*, "Either more or less than two-thirds of the army are abroad." With regard to the contrary we are in a certain difficulty ; for we may as it were proceed in two directions, and take our choice between "All the army are abroad" and "None of the army are abroad." I hardly see on what principle we are to choose between these.

Fortunately, contrary opposition, unlike contradictory opposition, is of very little logical importance.

59. The Opposition of Singular Propositions.

Take the proposition, Socrates is wise. The contra-

dictory is—Socrates is not wise ; and so long as we keep to the same terms, we cannot go beyond this simple denial. We have therefore no contrary distinct from the contradictory. This opposition of singulars has been called *secondary contradiction* (Mansel's *Aldrich*, p. 56).

There are indeed two methods of treatment according to which we might find a distinct contrary and contradictory in the case of singular propositions, but I think that the above treatment according to which they are not distinguished is preferable to either.

(1) We might introduce the material contrary of the predicate instead of its mere contradictory, (compare section 28). Thus we should have—

Original proposition, Socrates is wise ;

Contradictory, Socrates is not wise ;

Contrary, Socrates has not a grain of sense.

This might be called the *material* contrary of the given proposition[1]. A fresh term is introduced that could not be formally obtained out of the given proposition. It still remains true that the singular proposition has no *formal* contrary distinct from its contradictory.

(2) Some principle of separation into parts might be introduced according to which the subject would be no longer a whole indivisible ; for example, Socrates might be regarded as having different characteristics at different times or under different conditions. The original proposition would then be read Socrates is always wise, and the contradictory would be Socrates is sometimes not wise, while the contrary would be Socrates is never wise. Treated in this manner, however, the proposition hardly remains a really singular proposition.

---

[1] The same distinction might be applied to general propositions.

60. Can the ordinary doctrine of the opposition of propositions be applied to hypothetical and disjunctive propositions?

It has been already shewn that the ordinary distinctions of quantity and quality may be applied to Hypothetical Propositions, and it follows that the ordinary doctrine of opposition will also apply to them. We have

If A is B, C is D. **A.**

In some cases in which A is B, C is D. **I.**

If A is B, C is not D. **E.**

In some cases in which A is B, C is not D. **O.**

Then, as in the case of Categoricals,—

A and **I**, **E** and **O** are subalterns.

A and **E** are contraries.

A and **O**, **E** and **I** are contradictories.

I and **O** are subcontraries.

There is more danger of contradictories being confused with contraries in the case of Hypotheticals than there is in the case of Categoricals. *If A is B, C is not D* is very liable to be given as the contradictory of *If A is B, C is D.* But it clearly is not its contradictory, *so far as they are general propositions*, since both may be false. For example, the two statements,—If the Times says one thing, the Pall Mall says another; If the Times says one thing, the Pall Mall says the same, *i.e.*, does not say another,—are both false: the two papers are sometimes in agreement and sometimes not.

If however the Hypothetical proposition is of the nature of a Singular, that is, if the thing referred to in the antecedent can happen but once; then as in the case of Singular Categorical propositions, the Contradictory and the Contrary are not to be distinguished. Taking the proposition—If I

perish in the attempt, I shall not die unavenged; its con-
tradictory may fairly be stated—If I perish in the attempt,
I shall die unavenged.

We cannot apply distinctions of quality to Disjunctives,
and therefore the ordinary doctrine of opposition cannot be
applied to them. We may however, find the contradictory
and the contrary of a disjunctive proposition, such as A is
either B or C. Its Contradictory is—In some cases A is
neither B nor C; its Contrary—A is neither B nor C.
We observe then that the contradictory and contrary of a
disjunctive are not themselves disjunctive. What has been
said with regard to Singular Hypotheticals also applies
mutatis mutandis to what may be called Singular Dis-
junctives.

A point to which our attention is called by the above is
that the relation of reciprocity that holds between contra-
dictories does not always hold between contraries. If the
proposition β is the contradictory of the proposition a, then
a is also the contradictory of β; but if δ is the contrary of a,
it does not necessarily follow that a is the contrary of δ.
Thus, we have seen that the contrary of "A is either B or
C" is "A is neither B nor C." The contrary of the latter
however is "A is both B and C," which is not the original
proposition over again[1].

61. How would you apply the terms contradictory
and contrary to the case of complex propositions:
e.g., He was certainly stupid; and, if not mad, either
miserably trained, or misled by bad companions? [V.]

The criterion of contradictories given in section 58,
may be applied to the case of complex propositions. For
example, take the complex proposition X *is both A and B*,

[1] Cf. also the Examples given in section 58.

(where X is a singular term). Regarded as a whole, this statement is evidently false if X fails to be either one or the other of A and B. It is also clear that it must either be both of them or it must fail to be at least one of them. We have then this pair of contradictories,—

X is both A and B;
X is either not A or not B.

Thus, what we may perhaps call a conjunctive is contradicted by a disjunctive, and *vice versa*.

Next take the rather more complex proposition—

X is A, *and either B or C*[1].

Its contradictory, following the above rule, is

X is either not A or neither B nor C.

Next take the proposition

X is Y; *and if it is not Z, it is either Q or R*[1].

It may be reduced to

X is Y; and either Z, Q or R,

and we at once get the contradictory

X is either not Y or neither Z, Q nor R.

It will be noticed that the last example chosen is equivalent to the one given in the question, the terms of the latter being translated into symbols. The required contradictory is therefore—

Either he was not stupid, or he was neither mad, miserably trained nor misled by bad companions.

The application of the term contrary to complex propositions is of less interest. We may however consider that we have the contrary of such a proposition when we deny every part of the statement. Thus the contrary of "X is

[1] I still assume that the subject of the proposition is a singular term.

both A and B" is "X is neither A nor B"; of "X is A and either B or C," "X is neither A, B nor C"; and of the given proposition, "He was neither stupid nor mad nor miserably trained nor misled by bad companions."

62. What is the precise meaning of the assertion that a proposition—say "All grasses are edible"—is false?   [Jevons, *Studies in Deductive Logic*, p. 116.]

Professor Jevons discusses at some length the point here raised, but I find myself quite unable to agree with what he says in connection with it.

He commences by giving an answer, which may be called the orthodox one, and which I should certainly hold to be the correct one. When I assert that a proposition is false, I mean that its contradictory is true. The given proposition is of the form A, and its contradictory is the corresponding **O** proposition,—Some grasses are not edible. When, therefore, I say that it is false that all grasses are edible, I mean that some grasses are not edible. Professor Jevons however continues, " But it does not seem to have occurred to logicians in general to inquire how far similar relations could be detected in the case of disjunctive and other more complicated kinds of propositions. Take, for instance, the assertion that 'all endogens are *all* parallel-leaved plants.' If this be false, what is true? Apparently that one or more endogens are not parallel-leaved plants, or else that one or more parallel-leaved plants are not endogens. But it may also happen that no endogen is a parallel-leaved plant at all. There are three alternatives, and the simple falsity of the original does not shew which of the possible contradictories is true."

In this statement, there appear to me to be two errors. In the first place, in saying that one or more endogens are

not parallel-leaved plants, we do not mean to exclude the possibility that no endogen is a parallel-leaved plant at all. Symbolically, Some S is not P does not exclude No S is P. The three alternatives are therefore at any rate reduced to the two first given. But in the second place, I think Professor Jevons is in error in regarding each of these alternatives by itself as a contradictory of the original proposition. The true logical contradictory is the affirmation of the truth of *one or other* of these alternatives. If the original complex proposition is false we certainly know that the new complex proposition limiting us to such alternatives is true.

The point at issue may be further illustrated by taking the proposition in question in a symbolic form. *All S is all P* is a complex proposition, resolvable into the form, *All S is P, and all P is S.* In my view, it has but one contradictory, namely, *Either some S is not P, or some P is not S.*[1] If either of these alternatives holds good, the original statement must in its entirety be false ; and on the other hand, if the latter is false, one at least of these alternatives must be true. Professor Jevons speaks as if Some S is not P were by itself a contradictory of All S is all P. But it is merely inconsistent with it. They may both be false. No doubt in ordinary speech *contradictory* frequently implies no more than " inconsistent with," and if Professor Jevons means that we should also use the term contradictory in this sense in Logic, the question becomes a verbal one. But he means more than this ; he seems to mean that in some cases we can find no proposition that must be true when a given proposition is false. And here I hold that he is wrong.

[1] The contradictory of " All S is all P" may also be expressed ".S and P are not coextensive."

If the original proposition is complex, its contradictory will in general be complex too, and possibly still more complex; but that might naturally be expected. Compare the two preceding sections, where several cases are worked out in detail.

The above will I think indicate how misleading is Professor Jevons's further statement,—" It will be shewn in a subsequent chapter that a proposition of moderate complexity has an almost unlimited number of contradictory propositions, which are more or less in conflict with the original. The truth of any one or more of these contradictories establishes the falsity of the original, but the falsity of the original does not establish the truth of any one or more of its contradictories." No doubt a complex proposition may yield an indefinite number of other propositions the truth of any one of which is *inconsistent with* its own. But it has only one logical *contradictory*, which contradictory as suggested above is likely to be a still more complex proposition affirming a number of alternatives one or other of which must hold if the original proposition is false.

With the point here raised Professor Jevons mixes up another, with regard to which his view is almost more misleading. He says, "But the question arises whether there is not confusion of ideas in the usual treatment of this ancient doctrine of opposition, and whether a contradictory of a proposition is not any proposition which involves the falsity of the original, but is not the sole condition of it. I apprehend that any assertion is false which is made without sufficient grounds. It is false to assert that the hidden side of the moon is covered with mountains, not because we can prove the contradictory, but because we know that the assertor must have made the assertion without evidence.

If a person ignorant of mathematics were to assert that 'all involutes are transcendental curves,' he would be making a false assertion, because, whether they are so or not, he cannot know it." Surely in Logic we cannot regard the truth or falsity of a proposition as depending upon the knowledge of the person who affirms it, so that the same proposition would now be true, now false. The question "What is truth?" may be an enormously difficult one to answer absolutely, and I need not say that I shall not attempt to deal with it here ; but unless we are allowed to proceed from the falsity of "All S is P" to the truth of "Some S is not P," I do not think we can go far in Logic.

63. Analyse all that is implied in the assertion of the falsity of each of the following propositions :—

(1)   Roger Bacon was a giant.

(2)   Descartes died before Newton was born.

(3)   Bare assertion is not necessarily the naked truth.

(4)   All kinds of grasses except one or two species are not poisonous.

[Jevons, *Studies*, p. 124.]

64. Assign precisely the meaning of the assertion that it is false to say that some English soldiers did not behave discreditably in South Africa.   [L.]

65. Examine in the case of each of the following propositions the precise meaning of the assertion that the proposition is false :—

(i)   Some electricity is generated by friction.

(ii) Oxygen and nitrogen are constituents of the air we breathe.

(iii) If a straight line falling upon two other straight lines make the alternate angles equal to each other, these two straight lines shall be parallel.

(iv) Actions are either good, bad, or indifferent.

CHAPTER III.

66. The meaning of logical *conversion*. The *ordinary conversion* of propositions.

By *Conversion*, in a broad sense, is meant a change in the position of the terms of a proposition [1].

Logic, however, is concerned with conversion only in so far as the truth of the new proposition obtained by the process is a legitimate inference from the truth of the original proposition. This is what Whately means when he says that " no conversion is employed for any logical purpose unless it be *illative*" (*Elements of Logic*, p. 74). For example, the change from All *S* is *P* to All *P* is *S* is not a logical conversion, since the truth of the latter proposition does not follow from the truth of the former.

The simplest form of logical conversion may be defined as follows, and it may be distinguished from other forms by being called *ordinary* conversion :—By *ordinary conversion* is meant *a process of immediate inference in which from a given proposition we infer another, having the predicate of the original proposition for subject, and its subject for predicate.*

---

[1] Ueberweg (Lindsay's translation, p. 294) defines Conversion thus. Compare also De Morgan, p. 58.

Thus, given a proposition having S for its subject and P for its predicate, we seek to obtain by immediate inference a new proposition having P for its subject and S for its predicate; and applying this rule to the four fundamental forms of proposition, we get the following table:—

Original Proposition.	Converse.
All S is P. **A.**	Some P is S. **I.**
Some S is P. **I.**	Some P is S. **I.**
No S is P. **E.**	No P is S. **E.**
Some S is not P. **O.**	(None.)

67.   Simple Conversion, and Conversion *per accidens*.

It will be observed that in the case of I and E, the converse is of exactly the same form as the original proposition (or *convertend*); we do not lose any part of the information given us by the convertend, and we can pass back to it by re-conversion of the converse. The convertend and its converse are *equivalent* propositions.   The conversion in both these cases is said to be *simple*.

In the case of **A**, it is different; although we start with a universal proposition, we obtain by conversion a particular one only, and by no means of operating upon the converse can we regain the original proposition. The convertend and its converse are not equivalent propositions.   This is called conversion *per accidens*[1], or conversion *by limitation*.

[1] The conversion of A is said by Mansel to be called conversion

68. Particular negative propositions do not admit of *ordinary* conversion.

It is clear that if the converse is to be a legitimate formal inference from the original proposition (or convertend), it must distribute no term that was not distributed in the convertend. From this it follows immediately that Some *S* is not *P* does not admit of ordinary conversion ; for *S* which is undistributed in the convertend would become the predicate of a negative proposition in the converse, and would therefore be distributed. (I may remind the reader that in what I have called ordinary conversion, with which alone we are now dealing, we do not admit the contradictory of either the original subject or the original predicate as one of the terms of our converse.)

I cannot understand why Professor Jevons should say that the fact that the particular negative proposition is incapable of ordinary conversion "constitutes a blot in the ancient logic" (*Studies in Deductive Logic*, p. 37). We shall find subsequently that just as much can be inferred from the particular negative as from the particular affirmative, (since the latter unlike the former does not admit of contraposition). Less can be inferred from either of them than can be inferred from the corresponding universal proposition, and this is obviously because the latter gives all the informa-

---

per accidens " because it is not a conversion of the universal *per se,* but by reason of its containing the particular. For the proposition ' Some *B* is *A*' is *primarily* the converse of ' Some *A* is *B*,' *secondarily* of ' All *A* is *B*'" (Mansel's *Aldrich*, p. 61). Professor Baynes seems to deny that this is the correct explanation of the use of the term (*New Analytic of Logical Forms*, p. 29) ; but however this may be, I do not think that we can really regard the converse of *A* as obtained through its subaltern. We proceed directly from "All *A* is *B*" to "Some *B* is *A*" without the intervention of "Some *A* is *B*."

tion given by the particular proposition and more beside. No logic, symbolic or other, can actually obtain more from the given information than the ancient logic does.

69. Give the converse of the following propositions :—

   (1)   A stitch in time saves nine.
   (2)   None but the brave deserve the fair.
   (3)   He can't be wrong whose life is in the right.
   (4)   The virtuous alone are happy.

No difficulty can be found in converting or performing other immediate inferences upon any given proposition if it is once brought into logical form, its quantity and quality being determined, and its subject, copula and predicate being definitely distinguished from one another.

If this rule is neglected, the most absurd results may be elicited. For example, amongst several curious converses of the first of the above propositions I have had seriously given,—Nine stitches save a stitch in time. Here it is of course entirely overlooked that "save" cannot be a logical copula. The proposition may be written, All stitches in time | are | things that save nine stitches. This being an **A** proposition is only convertible *per accidens*, thus, Some things that save nine stitches are stitches in time. The following is wrong,—The means of saving nine stitches is a stitch in time ; since there may be other ways of saving nine.

"None but the brave deserve the fair." For the converse of this I have had,—The fair deserve none but the brave ; and, again, No one ugly deserves the brave. Logically the proposition may be written, No one who is not brave is deserving of the fair. This, being an **E** proposition, may

be converted simply, giving, No one deserving of the fair is not brave.

"He can't be wrong whose life is in the right." Written in strict logical form, this proposition becomes,—No one whose life is in the right is able to be in the wrong; and therefore its converse is,—No one who is able to be in the wrong is one whose life is in the right. This proposition may now be written in the more natural but not strictly logical form, His life cannot be in the right who can himself be wrong.

"The virtuous alone are happy." In logical form this may be written either, No one who is not virtuous is happy, or All who are happy are virtuous. Taking it in the first form, the converse is—No one who is happy is not virtuous; and from this we may again get the second form by changing its quality[1]—All who are happy are virtuous. The converse of this is,—Some who are virtuous are happy.

70. State in logical form and convert the following propositions :—

(1) There's not a joy the world can give like that it takes away.

(2) He jests at scars who never felt a wound.

(3) Axioms are self-evident.

(4) Natives alone can stand the climate of Africa.

(5) Not one of the Greeks at Thermopylæ escaped.

(6) All that glitters is not gold.   [O.]

71. Give all the logical opposites of the proposition :—Some rich men are virtuous ; and also the

---

[1] Cf. section 73.

converse of the contrary of its contradictory.   How is the latter directly related to the given proposition ?

72.   Point out any possible ambiguities in the following propositions, and shew the importance of clearing up such ambiguities for logical purposes :—

(i)   Some of the candidates have been successful.

(ii)   Either some gross deception was practised or the doctrine of spiritualism is true.

(iii)   All are not happy that seem so.

(iv)   All the fish weighed five pounds.

Give the contradictory and (where possible) the converse of each of these propositions.

CHAPTER IV.

THE OBVERSION AND CONTRAPOSITION OF PROPOSITIONS.

73. The Obversion of Propositions.

Obversion is *the process of changing the quality of a proposition without altering its meaning.* This change of quality may always be made *if at the same time we substitute for the predicate its contradictory.*

Applying this rule, we have the following table :—

Original Proposition.	Obverse.
All *S* is *P*. **A.**	No *S* is not-*P*. **E.**
Some *S* is *P*. **I.**	Some *S* is not not-*P*. **O.**
No *S* is *P*. **E.**	All *S* is not-*P*. **A.**
Some *S* is not *P*. **O.**	Some *S* is not-*P*. **I.**

The term *Obversion* is used by Professor Bain, and it is a convenient one. The process is also called *Permutation* (Fowler), *Aequipollence* (Ueberweg), *Infinitation* (Bowen), *Immediate Inference by Privative Conception* (Jevons), *Contraversion* (De Morgan), *Contraposition* (Spalding).

Obversion depends on the supposition that two negatives make an affirmative. De Morgan (*Formal Logic*, pp. 3, 4) points out that in ordinary speech this is not always strictly true. For example, "not unable" is scarcely used as strictly equivalent to "able," but is understood to imply a somewhat lower degree of ability. "John is able to translate Virgil" is taken to mean that he can translate it well; "Thomas is not unable to translate Virgil" is taken to mean that he can translate it—indifferently. This distinction, however, depends a good deal on the accentuation of the sentence; and it is not one of which Logic can take account. Logically, "*A*" and "not not-*A*" must be regarded as strictly equivalent.

74. Formal Obversion distinguished from Material Obversion.

By *Formal Obversion* is meant the kind of obversion discussed in the previous section, and I think that it is the only kind of obversion that Formal Logic need recognise.

Professor Bain uses the expression *Material Obversion*, and by it he means the process of making "Obverse Inferences which are justified only on an examination of the matter of the proposition" (*Logic*, i. p. 111). He gives as examples,—"Warmth is agreeable; therefore, cold is disagreeable. War is productive of evil; therefore, peace is productive of good. Knowledge is good; therefore, ignorance is bad." I should be inclined to doubt whether these are legitimate inferences, formal or otherwise. The conclusions would appear to require quite independent investigations to establish them. For example, granted that warmth is agreeable, it might be that every other state of temperature is agreeable also.

75. Give the obverse of the following proposi-
tions :—

(1) Whatever is, is right.

(2) No news is good news.

(3) Good orators are not always good statesmen.

(4) A stitch in time saves nine.

76. Conversion by Contraposition.

Contraposition (also called *Conversion by Negation*) is *a
process of immediate inference in which from a given proposi-
tion we infer another proposition having the contradictory of
the original predicate for its subject, and the original subject
for its predicate*[1].

---

[1] There is some difference between logicians as to whether the con-
trapositive of All *S* is *P* is *No not-P is S* or *All not-P is not-S*. It is
merely a verbal question, depending on our original definition of contra-
position. It will be observed that All not-*P* is not-*S* is the obverse of
No not-*P* is *S*, and if we regard All not-*P* is not-*S* as the contrapositive
of All *S* is *P*, our definition of contraposition must be altered to—"a
process of immediate inference in which from a given proposition we
infer another proposition having the contradictory of the original pre-
dicate for its subject, and the *contradictory* of the original subject for its
predicate." In this case, what I have originally defined as contra-
position may be called conversion by negation. Careful note should be
taken of this difference of usage, and then no difficulty is likely to
result. Taking the following definition, we might call either form a
contrapositive of the original proposition,—"contraposition is a process
of immediate inference in which from a given proposition we infer an-
other proposition having the contradictory of the original predicate for
its subject." It is here left an open question whether the predicate of
the contrapositive is to be the original subject or the contradictory of
the original subject.

The following is from Mansel's *Aldrich*, p. 61,—"Conversion by
contraposition, which is not employed by Aristotle, is given by Boethius

Thus, given a proposition having *S* for its subject and *P* for its predicate, we seek to obtain by immediate inference a new proposition having not-*P* for its subject and *S* for its predicate.

From the definition we can immediately deduce the following rule for obtaining the contrapositive of a given proposition :—*Obvert the original proposition, and then convert the proposition thus obtained.*   For given a proposition with *S* for subject and *P* for predicate, obversion will yield a new proposition with *S* for subject and not-*P* for predicate, and the conversion of this will make not-*P* the subject and *S* the predicate ;   *i.e.*, we shall have found the contrapositive of the given proposition.

in his first book, *De Syllogismo Categorico*. He is followed by Petrus Hispanus. It should be observed, that the old Logicians, following Boethius, maintain, that in conversion by contraposition, as well as in the others, the *quality* should remain unchanged. Consequently the converse of 'All *A* is *B*' is 'All not-*B* is not-*A*', and of 'Some *A* is not *B*,' 'Some not-*B* is not not-*A*.'   It is simpler, however, to convert *A* into *E* and *O* into *I* ('No not-*B* is *A*'; 'Some not-*B* is *A*') as is done by Wallis and Abp. Whately ; and before Boethius by Apuleius and Capella, who notice the conversion, but do not give it a name. The principle of this conversion may be found in Aristotle, *Top.* II. 8. 1, though he does not employ it for logical purposes."

In most text-books, no *definition* of contraposition is given at all, and it may be pointed out that in the attempt to generalise from special examples, Jevons in his *Elementary Lessons in Logic* gets into difficulties. For the contrapositive of *A* he gives All not-*P* is not-*S*; *O* he says has no contrapositive, (but only a converse by negation, Some not-*P* is *S*) ; and for the contrapositive of *E* he gives No *P* is *S*. I have failed to discover that any precise meaning can be attached to contraposition, according to which these results are obtainable.

It should be observed that if in contraposition the quality of the proposition is to remain unchanged as in Jevons's contrapositive of *A*, then the contrapositive both of *E* and *O* will be Some not-*P* is not not-*S*.

Applying this rule, we have the following table:—

Original Proposition.	Obverse.	Contrapositive.
All *S* is *P*. **A.**	No *S* is not-*P*. **E.**	No not-*P* is *S*. **E.**
Some *S* is *P*. **I.**	Some *S* is not not-*P*. **O.**	(None.)
No *S* is *P*. **E.**	All *S* is not-*P*. **A.**	Some not-*P* is *S*. **I.**
Some *S* is not *P*. **O.**	Some *S* is not-*P*. **I.**	Some not-*P* is *S*. **I.**

It is easy to shew that *Some S is P* has no contrapositive; for when it is obverted, it becomes a particular negative; but particular negatives do not admit of *ordinary* conversion, which is the process that must succeed obversion in order that a contrapositive may be arrived at.

It may be helpful if we here sum up the immediate inferences that have been obtained up to this point, making use of the symbols explained in section 38, and denoting not-*S* by *S'*, not-*P* by *P'*:—

Original Proposition.	Converse.	Obverse.	Contrapositive[1].	Obverted[1] Contrapositive.
SaP	PiS	SeP'	$P'eS$	$P'aS'$
SiP	PiS	SoP'		
SeP	PeS	SaP'	$P'iS$	$P'oS'$
SoP		SiP'	$P'iS$	$P'oS'$

[1] It should be remembered, as explained in the preceding note,

It will be shewn presently how this table of Immediate Inferences may be expanded.

With regard to the utility of the investigation as to what contrapositives are logically inferrible from given propositions, the following may be quoted from De Morgan:—

"The uneducated acquire easy and accurate use of the very simplest cases of transformation of propositions and of syllogisms. The educated, by a higher kind of practice, arrive at equally easy and accurate use of some more complicated cases: but not of all those which are treated in ordinary logic. Euclid may have been ignorant of the identity of 'Every X is Y' and 'Every not-Y is not-X,' for anything that appears in his writings: he makes the one follow from the other by a new proof each time" (*Syllabus*, p. 32).

77. How is Obversion related to Conversion by Negation or Contraposition?

Give the obverse and the contrapositive of the following propositions:—

    (*a*)   All animals feed;

    (*b*)   No plants feed;

    (*c*)   Only animals feed.   [L.]

78. Give the contrapositive of the following propositions:—

    (1)   A stitch in time saves nine.

    (2)   None but the brave deserve the fair.

    (3)   He can't be wrong whose life is in the right.

    (4)   The virtuous alone are happy.

that what is called the contrapositive above is sometimes called the converse by negation, and what is called the obverted contrapositive above is sometimes simply called the contrapositive.

79. Explain the nature of Conversion and Contraposition by reference to the following propositions :—

All associations are separable.

There are volcanoes which are never at rest. [v.]

80. " The angles at the base of an isosceles triangle are equal."

What can be inferred from this proposition by Obversion, Conversion, and Contraposition, without any appeal to geometrical proof?   [L.]

81. Transform the following propositions in such a way that, without losing any of their force, they may all have the same subject and the same predicate :—

No not-P is S,
All P is not-S,
Some P is S,
Some not-P is not not-S.

This problem may be briefly solved as follows :—

No not-P is S = No S is not-P = All S is P.

All P is not-S = No P is S = No S is P.

Some P is S = Some S is P.

Some not-P is not not-S = Some not-P is S

      = Some S is not-P = Some S is not P.

82. Describe the logical relations, if any, between each of the following propositions, and each of the others :—

(i)   There are no inorganic substances which do not contain carbon ;

(ii)   All organic substances contain carbon ;

(iii)   Some substances not containing carbon are organic ;

(iv)   Some inorganic substances do not contain carbon.   [C.]

This question can be most satisfactorily answered by reducing the propositions to such forms that they all have the same subject and the same predicate.

83. The application of the doctrines of Conversion and Contraposition to Hypothetical Propositions.

In a hypothetical proposition the antecedent and the consequent correspond respectively to the subject and the predicate of a categorical proposition. In Conversion therefore the old consequent must be the new antecedent, and in Contraposition the denial of the old consequent must be the new antecedent. Proceeding as before, this gives us immediately the following table :—

Original Proposition.	*Converse.*	*Contrapositive.*
If *A* is *B*, *C* is *D*. **A.**	In some cases in which *C* is *D*, *A* is *B*. **I.**	If *C* is not *D*, *A* is not *B*. **E.**
In some cases in which *A* is *B*, *C* is *D*. **I.**	In some cases in which *C* is *D*, *A* is *B*. **I.**	None.
If *A* is *B*, *C* is not *D*. **E.**	If *C* is *D*, *A* is not *B*. **E.**	In some cases in which *C* is not *D*, *A* is *B*. **I.**
In some cases in which *A* is *B*, *C* is not *D*. **O.**	None.	In some cases in which *C* is not *D*, *A* is *B*. **I.**

It must be remembered that we regard the quality of a hypothetical proposition as determined by the quality of the consequent.

The obverse of a hypothetical proposition is usually awkward to express. We may however find it if required; *e.g.*, the obverse of " If *A* is *B*, *C* is *D* " is "If *A* is *B*, *C* is not not-*D*."

84. Give the converse and the contrapositive of " If a straight line falling upon two other straight lines make the alternate angles equal to one another, these two straight lines shall be parallel."   [L.]

The application of the doctrines of Conversion and Contraposition to Hypothetical Propositions may be illustrated by means of the above proposition. We must note carefully that it is a universal affirmative, and is therefore only convertible *per accidens*. This is a point particularly liable to be overlooked where a universal converse can be legitimately inferred (as in the case of the above proposition), though not as an *immediate inference*. We are in no danger of saying, All men are animals, therefore, all animals are men; but we may be in danger of saying, All equilateral triangles are equiangular, therefore, all equiangular triangles are equilateral. From the point of view however of Formal Logic the latter inference is as erroneous as the former.

So far as the given proposition is concerned, we have—

Converse, In some cases in which two straight lines are parallel, a straight line falling upon them shall make the alternate angles equal to one another.

Contrapositive, If two straight lines are not parallel, then a straight line falling upon them shall make the alternate angles not equal to one another.

85. Give the contradictory, the contrary, the converse, and the contrapositive of the following propositions :

(1) Things equal to the same thing are equal to one another.

(2) No one is a hero to his valet.

(3) If there is no rain the harvest is never good.

(4) None think the great unhappy but the great.

(5) Fain would I climb but that I fear to fall.

86. Name the form of each of the following propositions ; and, where possible, give the converse and the contrapositive of each :—

(i) Some death is better than some life.

(ii) The candidates in each class are not arranged in order of merit.

(iii) Honesty is the best policy.

(iv) Not all that tempts your wand'ring eyes
And heedless hearts is lawful prize.

(v) If an import duty is a means of revenue, it does not afford protection.

(vi) Great is Diana of the Ephesians.

(vii) All these claims upon my time overpower me.

CHAPTER V.

87. In what cases can we obtain by immediate inference from a given proposition a new proposition having the contradictory of the original subject for its subject, and the original predicate for its predicate ?

A new form of immediate inference is here indicated, by which given a proposition having S for its subject and P for its predicate, we seek to obtain a new proposition having not-S for its subject and P for its predicate.

If such a proposition can be obtained at all, it will be by a certain combination of the elementary processes of ordinary conversion and obversion. We will take each of the fundamental forms of proposition and see what can be obtained (1) by first converting it, and then performing alternately the operations of obversion and conversion ; (2) by first obverting it, and then performing alternately the operations of conversion and obversion. We shall find that in each case we can go on till we reach a particular negative proposition whose turn it is to be converted.

(1) The results of performing the processes of conversion and obversion alternately, commencing with the *former*, are as follows :—

(i)   All *S* is *P*,

   therefore (by conversion), Some *P* is *S*,

   therefore (by obversion), Some *P* is not-*S*.

Here comes the turn for conversion; but we have an **O** proposition, and can therefore proceed no further.

(ii)   Some *S* is *P*,

   therefore (by conversion), Some *P* is *S*,

   therefore (by obversion), Some *P* is not-*S*;

and we can get no further.

(iii)   No *S* is *P*,

   therefore (by conversion), No *P* is *S*,

   therefore (by obversion), All *P* is not-*S*,

   therefore (by conversion), *Some not-S is P*,

   therefore (by obversion), Some not-*S* is not not-*P*.

In this case the proposition in italics is the immediate inference that was sought.

(iv)   Some *S* is not *P*.

In this case we are not able even to commence our series of operations.

(2)   The results of performing the processes of conversion and obversion alternately, commencing with the *latter*, are as follows :—

(i)   All *S* is *P*,

   therefore (by obversion), No *S* is not-*P*,

   therefore (by conversion), No not-*P* is *S*,

   therefore (by obversion), All not-*P* is not-*S*,

   therefore (by conversion), Some not-*S* is not-*P*,

   therefore (by obversion), *Some not-S is not P*.

Here again we have obtained the desired form.

(ii)   Some *S* is *P*,

   therefore (by obversion), Some *S* is not not-*P*.

(iii) No *S* is *P*,

therefore (by obversion), All *S* is not-*P*,

therefore (by conversion), Some not-*P* is *S*,

therefore (by obversion), Some not-*P* is not not-*S*.

(iv)   Some *S* is not *P*,

therefore (by obversion), Some *S* is not-*P*,

therefore (by conversion), Some not-*P* is *S*,

therefore (by obversion), Some not-*P* is not not-*S*.

We can now answer the question with which we commenced this enquiry. The required proposition can be obtained only if the given proposition is universal; we then have, according as it is affirmative or negative,—

All *S* is *P*, therefore, Some not-*S* is not *P*;

No *S* is *P*, therefore, Some not-*S* is *P*[1].

It must be observed that in the case of the former of these we commenced with obversion in order to get the new form, in the latter we commenced with conversion.

This form of immediate inference has been more or less casually recognised by various logicians; but I do not remember that it has ever received any distinctive name. Sometimes it has been vaguely classed under contraposition, (compare Jevons, *Elementary Lessons in Logic*, pp. 185, 6), but it is really as far removed from the process to which that designation has been given as the latter is from ordinary conversion. I venture to suggest the terms *Inversion* and *Inverse*[2]. Thus, *Inversion* is *a process of immediate inference*

---

[1] For assumptions respecting "existence" involved in these inferences, see chapter 8.

[2] Professor Jevons (carrying out a suggestion of Professor Robertson's) has introduced the term *Inverse* in a different sense. I do not however think that for logical purposes we want any new term in the sense in which he uses it; and I have been unable to think of any other equally suitable term for my own purpose, for which a new term really

*in which from a given proposition we infer another proposition
having the contradictory of the original subject for its subject,*

is needed, if the scheme of immediate inferences by means of conver-
sion and obversion is to be made scientifically complete. The term
contraverse has occurred to me, but I do not like it so well ; and this
again has been appropriated by De Morgan in another sense.

Professor Jevons's nomenclature is explained in the following passage
from his *Studies in Deductive Logic*, p. 32 :—" It appears to be indis-
pensable to endeavour to introduce some fixed nomenclature for the
relations of propositions involving two terms. Professor Alexander
Bain has already made an innovation by using the term *obverse*, and
Professor Hirst, Professor Henrici and other reformers of the teaching
of geometry have begun to use the terms converse and obverse in
meanings inconsistent with those attached to them in logical science
(*Mind*, 1876, p. 147). It seems needful, therefore, to state in the most
explicit way the nomenclature here proposed to be adopted with the
concurrence of Professor Robertson.

Taking as the original proposition ' All *A* are *B*,' the following are
what we may call the *related propositions—*

Inferrible.

Converse.   Some *B* are *A*.

Obverse.   No *A* are not *B*.

Contrapositive.   No not *B* are *A*, or, all not *B* are not *A*.

Non-Inferrible.

Inverse.   All *B* are *A*.

Reciprocal.   All not *A* are not *B*.

It must be observed that the converse, obverse, and contrapositive
are all true if the original proposition is true. The same is not neces-
sarily the case with the *inverse* and *reciprocal*. These latter two names
are adopted from the excellent work of Delbœuf, *Prolégomènes Philo-
sophiques de la Géométrie*, pp. 88—91, at the suggestion of Professor
Croom Robertson (*Mind*, 1876, p. 425)."

In this scheme what I propose to call the *Inverse* is not recognised
at all. On the other hand, I hardly see why the *non-inferrible* forms
need such a distinct logical recognition as is implied by giving them
distinct names ; while except in books on Logic I anticipate that the
term *converse* is likely still to be used in its non-logical sense, (*i.e.*, " All
B are *A*" is likely still to be spoken of as the converse of " All *A* are

and the original predicate for its predicate. In other words, given a proposition having S for subject and P for predicate, we obtain by inversion a new proposition having not-S for subject and P for predicate.

We may now sum up the results that have been obtained with regard to immediate inferences. Given two terms S and P, and admitting their contradictories not-S and not-P, we have eight possible forms of proposition as shewn in the following scheme:—

	Subject.	*Predicate.*
(i)	S	P
(ii)	S	not-P
(iii)	P	S
(iv)	P	not-S
(v)	not-P	S
(vi)	not-P	not-S
(vii)	not-S	P
(viii)	not-S	not-P

B"). It may be noted that in Jevons's use of terms, the inverse would be the same as the converse in the case of **E** and **I** propositions. I imagine also that in consistency there should be yet another term to express the relation of "No not-B is not-A" or "All not-B is A" to "No A is B"; it is, in the sense in which Jevons uses these terms, neither the Converse, Obverse, Contrapositive, Inverse nor Reciprocal.

These propositions may be designated respectively:—

   (i)   The original proposition,
   (ii)   The obverse,
   (iii) The converse,
   (iv) The obverted converse,
   (v)   The contrapositive,
   (vi) The obverted contrapositive,
   (vii) The inverse,
  (viii) The obverted inverse.

It has been shewn, in sections 66, 73, 76, and in the above, that if the original proposition is universal, we can infer from it propositions of all the remaining seven forms; but if it is particular, we can infer only three others.

Working out the different cases in detail we have:—

A.   (i)   Original proposition, *All S is P.*
      (ii)   Obverse, *No S is not-P.*
      (iii) Converse, *Some P is S.*
      (iv) Obverted converse, *Some P is not not-S.*
      (v)   Contrapositive, *No not-P is S.*
      (vi) Obverted Contrapositive, *All not-P is not-S.*
      (vii) Inverse, *Some not-S is not P.*
     (viii) Obverted Inverse, *Some not-S is not-P.*

I.   (i)   Original proposition, *Some S is P.*
      (ii)   Obverse, *Some S is not not-P.*
      (iii) Converse, *Some P is S.*
      (iv) Obverted Converse, *Some P is not not-S.*
      (v)   Contrapositive, none can be inferred.
      (vi) Obverted Contrapositive, none.
      (vii) Inverse, none.
     (viii) Obverted Inverse, none.

E.   (i)   Original proposition, *No S is P.*
      (ii)   Obverse, *All S is not-P.*

     (iii)   Converse, *No P is S.*

     (iv)   Obverted Converse, *All P is not-S.*

     (v)   Contrapositive, *Some not-P is S.*

   (vi)   Obverted Contrapositive, *Some not-P is not not-S.*

    (vii)   Inverse, *Some not-S is P.*

  (viii)   Obverted Inverse, *Some not-S is not not-P.*

O.   (i)   Original proposition, *Some S is not P.*

     (ii)   Obverse, *Some S is not-P.*

    (iii)   Converse, none can be inferred.

     (iv)   Obverted Converse, none.

     (v)   Contrapositive, *Some not-P is S.*

   (vi)   Obverted Contrapositive, *Some not-P is not not-S.*

    (vii)   Inverse, none.

  (viii)   Obverted Inverse, none.

All the above is summed up in the following Table (using the symbols described in section 38, and denoting not-*S* by *S'*, not-*P* by *P'*):—

		A.	I.	E.	O.
i	Original Proposition	SaP	SiP	SeP	SoP
ii	Obverse	SeP'	SoP'	SaP'	SiP'
iii	Converse	PiS	PiS	PeS	
iv	Obverted Converse	PoS'	PoS'	PaS'	
v	Contrapositive	$P'eS$		$P'iS$	$P'iS$
vi	Obverted Contrapositive...	$P'aS'$		$P'oS'$	$P'oS'$
vii	Inverse	$S'oP$		$S'iP$	
viii	Obverted Inverse............	$S'iP'$		$S'oP'$	

It is worth noticing that we can infer the same number of propositions from **E** as from **A** (7), from **O** as from **I** (3), and the same number of universal propositions from **E** as from **A** (3); also in two cases we can get no more from **A** than from **I**, and no more from **E** than from **O**.

88. Give the inverse of the following propositions :—

(1) A stitch in time saves nine.

(2) None but the brave deserve the fair.

(3) He can't be wrong whose life is in the right.

(4) The virtuous alone are happy.

89. Assuming that no organic beings are devoid of Carbon, what can we thence infer respectively about beings which are not organic, and things which are not devoid of carbon ?   [L.]

90. Make as many Immediate Inferences as you can from the following propositions :—

(1) Civilization and Christianity are coextensive.

(2) Uneasy lies the head that wears a crown.

(3) Your money or your life !   [L.]

91. Write out all the propositions that must be true, and all that must be false, if we grant that

(α) A straight line is the shortest distance between two points ;

(β) All the angles of a triangle are equal to two right angles ;

(γ) Not all the great are happy.   [C.]

92. De Morgan says (*Fourth Memoir on the Syllogism*, p. 5) of the Laws of Thought : " Every transgression of these laws is an invalid inference ; every valid inference is not a transgression of these laws. But I cannot admit that everything which is not a transgression of these laws is a valid inference." Investigate the logical relations between these three assertions.   [Jevons, *Studies*, p. 301.]

93. Assign the logical relation, if any, between each pair of the following propositions :—

(1)   All crystals are solids.

(2)   Some solids are not crystals.

(3)   Some not crystals are not solids.

(4)   No crystals are not solids.

(5)   Some solids are crystals.

(6)   Some not solids are not crystals.

(7)   All solids are crystals.   [L.]

94. " All that love virtue love angling."

Arrange the following propositions in the four following groups :—

(α)   Those which can be inferred from the above proposition ;

(β)   Those from which it can be inferred ;

(γ)   Those which do not contradict it, but which cannot be inferred from it ;

(δ)   Those which contradict it.

    (i) None that love not virtue love angling.

   (ii) All that love angling love virtue.

  (iii) All that love not angling love virtue.

  (iv) None that love not angling love virtue.

   (v) Some that love not virtue love angling.

  (vi) Some that love not virtue love not angling.

 (vii) Some that love not angling love virtue.

(viii) Some that love not angling love not virtue.

CHAPTER VI.

THE DIAGRAMMATIC REPRESENTATION OF PROPOSITIONS.

95. Methods of illustrating the ordinary processes of Formal Logic by means of Diagrams.

Representing the individuals included in any class, or denoted by any name, by a circle, it will be obvious that the five following diagrams represent all possible relations between any two classes:—
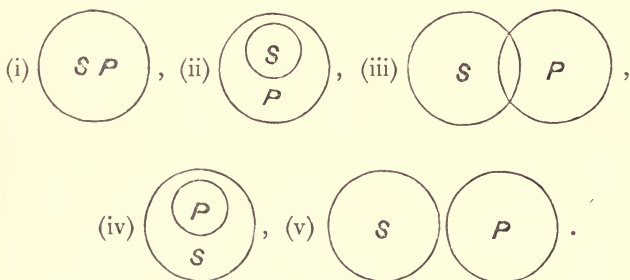
The force of the different propositional forms is to exclude one or more of these possibilities[1].

[1] The method of interpreting a proposition by what it excludes or negatives is discussed in more detail in chapter VIII.

All S is P limits us to

S P   or   S P .
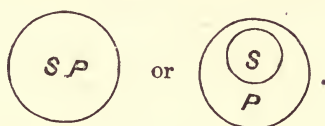
Some S is P to one of the four

S P ,   S P ,   S P ,

P S .

No S is P to

S P .
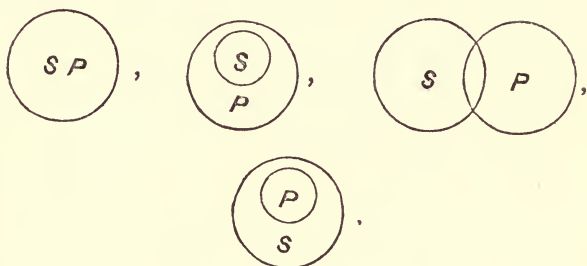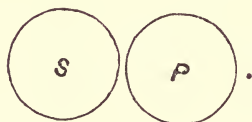
Some S is not P to one of the three

S P ,   P S ,   S P .
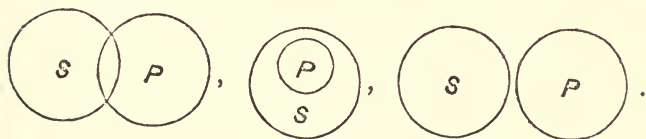
To represent All *S* is *P* by a single diagram, thus

S P ,
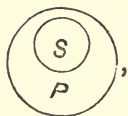
or Some *S* is *P* by a single diagram, thus
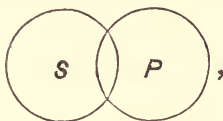
is most misleading; since in each case the proposition really
leaves us with other alternatives. This method of employ-
ing the diagrams is however adopted by most logicians who
have used them, including Sir William Hamilton (*Logic*, i.
p. 255), and Professor Jevons (*Elementary Lessons in Logic*,
pp. 72—75); and the attempt at such simplification has
brought their use into undeserved disrepute. Thus, Mr
Venn remarks, "The common practice, adopted in so many
manuals, of appealing to these diagrams,—Eulerian diagrams
as they are often called,—seems to me very questionable.
The old four propositions **A, E, I, O**, do not exactly corre-
spond to the five diagrams, and consequently none of the
moods in the syllogism can in strict propriety be represented
by these diagrams." (*Symbolic Logic*, p. 15, compare also pp.
424, 425). This is undoubtedly sound as against the use
of Euler's circles by Hamilton and Jevons; but I do not
admit its force as against their use in the manner described
above[1]. Many of the operations of Formal Logic can be
satisfactorily illustrated by their aid; though it is true that
they become somewhat cumbrous in relation to the Syllogism.
Thus, they may be employed,—(1) To illustrate the distri-
bution of the predicate in a proposition. In the case of each
of the four fundamental propositions we may shade the part
of the predicate concerning which knowledge is given us.

We then have,—
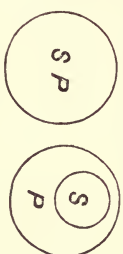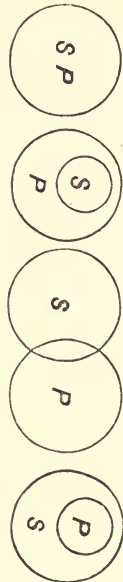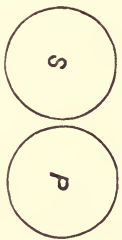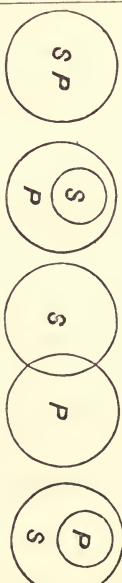
[1] They are used correctly by Ueberweg. Cf. Lindsay's translation
of Ueberweg's *System of Logic*, pp. 216—218.
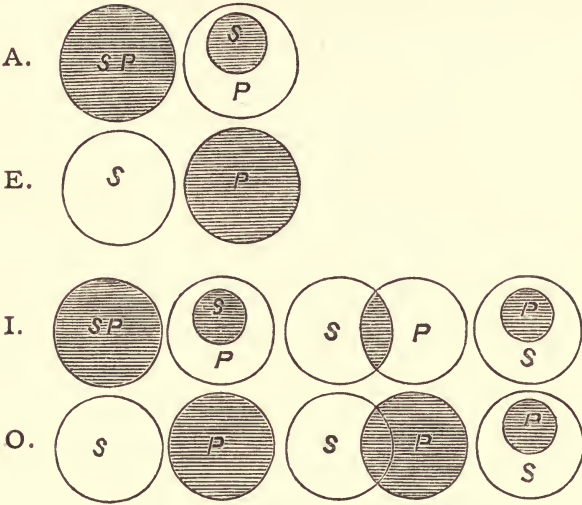
The result is that with **A** and **I** there are cases in which only part of P is shaded; whereas with **E** and **O**, the whole of P is in every case shaded; and it is made clear that negative propositions distribute, while affirmative propositions do not distribute their predicates.

(2) To illustrate the Opposition of Propositions. Comparing two contradictory propositions, *e.g.*, **A** and **O**, we see that they have no case in common, but that between them they exhaust all possible cases. Hence the truth, that two contradictory propositions cannot be true together but that one of them must be true, is brought home to us under a new aspect. Again, comparing two subaltern propositions, *e.g.*, **A** and **I**, we notice that the former gives us all the information given by the latter and something more, since it still further limits the possibilities.

To make this point the more clear the following table is appended:—

	Relations to which the proposition limits us.	Relations definitely excluded.
A.	SP; S P	S P; S P; S P
I.	SP; S P; S P; S P	S P
E.	S P	SP; S P; S P; S P
O.	S P; S P; S P; S P	SP; S P

(3) To illustrate the Conversion of Propositions. Thus, it is made quite clear how it is that **A** admits only of Conversion *per accidens*. *All S is P* limits us to one or other of the following
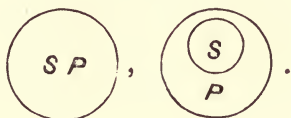
The problem of Conversion is—What do we know of P in either case? In the first, we have All P is S, but in the second Some P is S; *i.e.*, taking the cases indifferently, we have Some P is S and nothing more.

Again, it is made clear how it is that **O** is inconvertible. *Some S is not P* limits us to one or other of the following,—
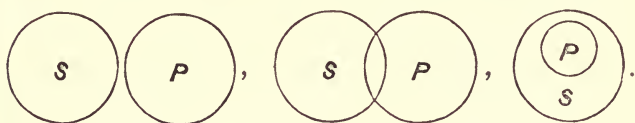
What then do we know concerning P? The three cases give us respectively

(i)   No P is S,

(ii) Some P is S, and Some P is not S,

(iii) All P is S.

(i) and (iii) are contraries, and (ii) is contradictory to both of them. Hence *nothing* can be affirmed of P that is true in all three cases indifferently.

(4) To illustrate the more complicated forms of immediate inference. Taking, for example, the proposition *All S is P*, we may ask, What does this enable us to assert about

not-P and not-S respectively? We have one or other of
these cases
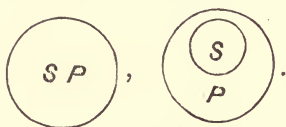
With regard to not-P, these yield respectively,

(i) No not-P is S,

(ii) No not-P is S. And thus we obtain the contraposi-
tive of the given proposition.

With regard to not-S we have

(i) All not-S is not-P,

(ii) Some not-S is not-P, (unless P constitutes the
entire universe of discourse, a point that is further discussed
subsequently); *i.e.*, in either case we may infer Some not-S
is not-P.   **E, I, O** may be dealt with similarly.

The application of the diagrams to syllogisms and to
special problems will be shewn in subsequent sections.

With regard to all the above, it may be said that the use
of the circles gives us nothing that could not easily have
been obtained independently. This is of course true; but
no one, who has had experience of the difficulty that is
sometimes found by students in really mastering the ele-
mentary principles of Formal Logic, and especially in deal-
ing with immediate inferences, will despise any means of
illustrating afresh the old truths, and presenting them under
a new aspect.

The fact that we have not a single diagram correspond-
ing to each fundamental form of proposition is fatal if we
wish to illustrate any complicated train of reasoning in this
way; but in indicating the real nature of the knowledge

given by the propositions themselves, it is rather an advantage as shewing how limited in some cases this knowledge actually is.

The diagrams invented and used by Mr Venn (*Symbolic Logic*, Chapter 5) are extremely interesting and valuable. In this scheme the diagram
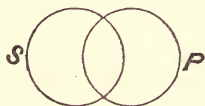
does not itself represent any proposition, but the framework into which propositions may be fitted. Denoting not-S by S' and what is both S and P by SP, &c., it is clear that everything must be contained in one or more of the four classes SP, SP', $S'P$, $S'P'$; and the above diagram shews four compartments, (one being that which lies outside both the circles), corresponding to these four classes. Every universal proposition denies the existence of one or more of such classes, and it may therefore be diagrammatically represented by shading out the corresponding compartment or compartments. Thus, *All S is P*, which denies the existence of SP', is represented by
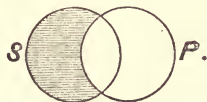
No S is P by
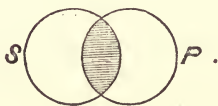
If we have three terms we have three circles and eight compartments, thus:—
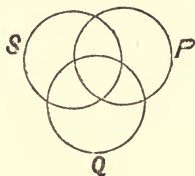
All S is P or Q is represented by
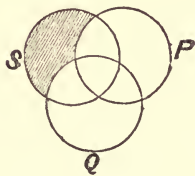
All S is P and Q by
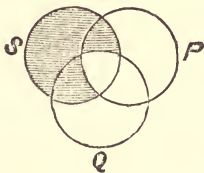
It is in cases involving three or more terms that the advantage of this scheme over the Eulerian scheme is most manifest. It is not however so easy to apply these diagrams to the case of particular propositions[1].

[1] " If we introduce particular propositions we must of course employ some additional form of diagrammatical notation......We might, for example, just draw a bar across the compartments declared to be saved; remembering of course that, whereas destruction is distributive, *i.e.*,

Lambert's scheme of representing propositions by combinations of straight lines will be touched upon in connection with the syllogism; compare section 180.

[A passing reference may be made to the fundamental objection raised by Mansel against the introduction of any such aids at all.   "If Logic is exclusively concerned with Thought, and Thought is exclusively concerned with Concepts, it is impossible to approve of a practice, sanctioned by some eminent Logicians, of representing the relation of terms in a syllogism by that of figures in a diagram.   To illustrate, for example, the position of the terms in Barbara, by a diagram of three circles, one within another, is to lose sight of the distinctive mark of a concept, that it cannot be presented to the sense, and tends to confuse the mental inclusion of one notion in the sphere of another, with the local inclusion of a smaller portion of space in a larger" (*Prolegomena Logica*, p. 55).   In answering this objection, it seems sufficient to point out that even conceptualist logicians must recognise and deal with the *extension* of concepts, and that the Eulerian diagrams make no pretence of representing the concepts themselves, but only their extension.]

96.   Illustrate the relation between **A** and **E**, and between **I** and **O** by means of the Eulerian diagrams.

every included sub-section is destroyed, the salvation is only alternative or partial, *i.e.*, we can only be sure that some of the included sub-sections are saved.   Thus, 'No x is y,' leading to destruction of xy, will destroy both xyz and $xy\bar{z}$, ($\bar{z}$ denoting not-z), "if z has to be taken account of.   But 'Some x is y', saving a part of xy, does not in the least indicate whether such part is xyz or $xy\bar{z}$......If it were worth while thus to illustrate complicated groups of propositions of the kind in question, it could, I fancy, be done with very tolerable success."   Venn in *Mind*, 1883, pp. 599, 600.

97. Illustrate the conversion of **I**, the contraposition of **O**, and the inversion of **E**, by means of the Eulerian diagrams.

98. To what extent, if any, may the processes of Immediate Inference be illustrated by means of Mr Venn's diagrams?

99. Any information given with respect to two terms limits the possible relations between them to one or more of the five following,—
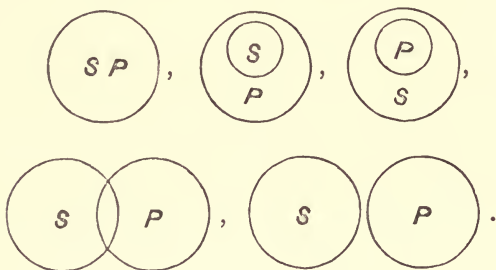
Shew how such information may in all cases be expressed by means of the propositional forms **A, I, E, O.**

Let the five relations be designated respectively α, β, γ, δ, ϵ[1]. Information is given when the possibility of one or more of these is denied; in other words, when we are limited to one, two, three, or four of them. Let limitation

[1] Thus, the terms being S and P, α denotes that S and P are wholly coincident; β that P contains S and more besides; γ that S contains P and more besides; δ that S and P overlap each other, but that each includes something not included by the other; ϵ that S and P have nothing whatever in common.

to a or β, (*i.e.*, the exclusion of γ, δ and ϵ), be denoted by a, β ; limitation to a, β or γ, (*i.e.*, the exclusion of δ and ϵ), by a, β, γ; and so on.

Now if we wish to express such information by means of the four ordinary propositional forms, we find that some-times a single proposition will suffice for our purpose; thus a, β is expressed by "All S is P." Sometimes we require a combination of propositions; thus a is expressed by saying that "All S is P, and also All P is S," (since All S is P excludes γ, δ, ϵ, and All P is S further excludes β). Some other cases are still more complicated; thus the fact that we are limited to a or δ cannot be expressed more simply than by saying "Either All S is P and All P is S, or else Some S is P, Some S is not P, and Some P is not S."

Let $\mathbf{A} =$ All S is P, $\mathbf{A}_1 =$ All P is S, and similarly for the other propositions. Also let $\mathbf{AA}_1 =$ All S is P and All P is S, &c. Then the following is a scheme for all possible cases :—

limitation to	denoted by	limitation to	denoted by
α	$\mathbf{AA}_1$	α, β, γ	$\mathbf{A}$ or $\mathbf{A}_1$
β	$\mathbf{AO}_1$	α, β, δ	$\mathbf{A}$ or $\mathbf{IO}_1$
γ	$\mathbf{A}_1\mathbf{O}$	α, β, ϵ	$\mathbf{A}$ or $\mathbf{E}$
δ	$\mathbf{IOO}_1$	α, γ, δ	$\mathbf{A}_1$ or $\mathbf{IO}$
ϵ	$\mathbf{E}$	α, γ, ϵ	$\mathbf{A}_1$ or $\mathbf{E}$
α, β	$\mathbf{A}$	α, δ, ϵ	$\mathbf{AA}_1$ or $\mathbf{OO}_1$
α, γ	$\mathbf{A}_1$	β, γ, δ	$\mathbf{IO}$ or $\mathbf{IO}_1$
α, δ	$\mathbf{AA}_1$ or $\mathbf{IOO}_1$	β, γ, ϵ	$\mathbf{AO}_1$ or $\mathbf{A}_1\mathbf{O}$ or $\mathbf{E}$
α, ϵ	$\mathbf{AA}_1$ or $\mathbf{E}$	β, δ, ϵ	$\mathbf{O}_1$
β, γ	$\mathbf{AO}_1$ or $\mathbf{A}_1\mathbf{O}$	γ, δ, ϵ	$\mathbf{O}$
β, δ	$\mathbf{IO}_1$	$\alpha, \beta, \gamma, \delta$	$\mathbf{I}$
β, ϵ	$\mathbf{AO}_1$ or $\mathbf{E}$	$\alpha, \beta, \gamma, \epsilon$	$\mathbf{A}$ or $\mathbf{A}_1$ or $\mathbf{E}$
γ, δ	$\mathbf{IO}$	$\alpha, \beta, \delta, \epsilon$	$\mathbf{A}$ or $\mathbf{O}_1$
γ, ϵ	$\mathbf{A}_1\mathbf{O}$ or $\mathbf{E}$	$\alpha, \gamma, \delta, \epsilon$	$\mathbf{A}_1$ or $\mathbf{O}$
δ, ϵ	$\mathbf{OO}_1$	$\beta, \gamma, \delta, \epsilon$	$\mathbf{O}$ or $\mathbf{O}_1$

It will be found that any other combinations of propositions than those given here involve either contradictions or redundancies, or else no information is given because all the five relations that are *a priori* possible still remain possible.

For example, **AI** is clearly redundant; **AO** is self-contradictory; **A** or **A₁O** is redundant (since the same informa-

tion is given by **A** or **A,**); **A** or **O** gives no information (since it excludes no possible case). The student is recommended to test other combinations similarly. It must be remembered that $I_1 = I$ and $E_1 = E$.

It should be noticed that if we read the first column downwards and the second column upwards we get pairs of contradictories.

CHAPTER VII.

100. Attempts to reduce immediate inferences to the mediate form[1].

Immediate inference is usually defined as the inference of a proposition from a single other proposition ; whereas mediate inference is the inference of a proposition from at least two other propositions.

(1) One of the old Greek Logicians, Alexander of Aphrodisias, establishes the conversion of **E** by means of a syllogism in *Ferio.*

<div align="center">

No S is P,

therefore, No P is S;

</div>

for if not, then by the law of contradiction, Some P is S; and we have this syllogism,—

<div align="center">

No S is P,

Some P is S,

therefore, Some P is not P,

</div>

a *reductio ad absurdum.*

---

[1] Students who have not already a technical knowledge of the syllogism may omit this section until they have read the earlier chapters of Part III.

Having proved the conversion of **E**, those of **A** and **I** will follow from it.

All S is P,

therefore, Some P is S;

for, if not, No P is S[1],

and therefore (by conversion) No S is P;

but this is inconsistent with the original supposition.

Similarly for **I**. (Compare Mansel's *Aldrich*, p. 62.)

(2) The contraposition of **A** may be established by means of a syllogism in *Camestres* as follows,—

Given   All S is P,

we have also   No not-P is P, by the law of contradiction,

therefore, No not-P is S.

(3) There is likewise an implicit syllogism in the following from Jevons, *Studies in Deductive Logic*, p. 44, "We may also prove the truth of the contrapositive (of the proposition All X is Y) indirectly; for what is not-Y must be either X or not-X; but if it be X it is by the premiss also Y, so that the same thing would be at the same time not-Y and also Y, which is impossible. It follows that we must affirm of not-Y the other alternative, not-X."

All the above are interesting, as illustrating the nature of immediate inferences; but regarded as proofs they labour under the disadvantage of deducing the less complex by means of the more complex.

I hardly know what is to be said in favour of the following :—

(4) Wolf obtains the subaltern of a universal proposition by a syllogism in *Darii*.

---

[1] This is itself an inference by Opposition.

Given   All S is P,

we have also   Some S is S, by the law of Identity,

therefore, Some S is P.

(Compare, Mansel, *Prolegomena Logica*, p. 217.)

(5) "Still more absurd is the elaborate system which Krug, after a hint from Wolf, has constructed, in which all immediate inferences appear as hypothetical syllogisms; a major premiss being supplied in the form, 'If all A is B, some A is B.' The author appears to have forgotten, that either this premiss is an additional empirical truth, in which case the immediate reasoning is not a logical process at all; or it is a formal inference, presupposing the very reasoning to which it is prefixed, and thus begging the whole question" (Mansel, *Prolegomena Logica*, p. 217).

101. How far can the legitimacy of the various processes of Immediate Inference be immediately deduced from the laws of Identity, Contradiction and Excluded Middle?

Law of Identity,—A is A.

Law of Contradiction,—A is not not-A.

Law of Excluded Middle,—A is either B or not-B.

We may consider the application of these laws to

(1)   inferences based on the square of opposition;

(2)   obversion;

(3)   conversion.

(1)   The inferences based on the square of opposition may be considered to depend exclusively on the above laws of thought.   For example, from the truth of All S is P we may infer the truth of Some S is P by the law of Identity, and the falsity of some S is not P by the law of Contradiction; from the falsity of All S is P we may infer

the truth of some S is not P by the law of Excluded Middle.

(2) Obversion also may be based entirely on the laws of Contradiction and Excluded Middle. From All S is P we get No S is not-P by the law of Contradiction; and from No S is P we get All S is not-P by the law of Excluded Middle.

(3) The case of Conversion is different; and I do not see how this process can be based exclusively on these three laws of thought. Mansel holds that it can, but so far as I am able to discover he makes no attempt to establish his position in detail. How, for example, would the application of the three laws of thought prove our inability to convert an **O** proposition? De Morgan appears to me to be perfectly justified in saying,—"When any writer attempts to shew *how* the perception of convertibility 'A is B gives B is A' follows from the principles of identity, difference and excluded middle, I shall be able to judge of the process: as it is, I find that others do not go beyond the simple assertion, and that I myself can detect the *petitio principii* in every one of my own attempts" (*Syllabus*, p. 47). The following attempt may be taken as a specimen :— " All A is B, therefore, some B is A; for if no B were A, then A would be both B and not B, which is impossible." It is clear however that conversion is already assumed in this reasoning.

If Conversion cannot be based exclusively on the three Laws of Thought, it follows that Contraposition and Inversion cannot be based exclusively on these Laws.

102. Proof of the various rules of Conversion.

The question as to what proof should be given of the various rules of conversion has been partially discussed in the two preceding sections. In them we discussed attempts

to prove conversions (1) by means of syllogisms, (2) by means of the three laws of identity, contradiction and excluded middle.

Bain writes as follows,—"When we examine carefully the various processes in Logic, we find them to be material to the very core. Take *Conversion.* How do we know that, if No X is Y, No Y is X? By examining cases in detail, and finding the equivalence to be true. Obvious as the inference seems on the mere formal ground, we do not content ourselves with the formal aspect. If we did, we should be as likely to say, All X is Y gives All Y is X; we are prevented from this leap merely by the examination of cases" (*Logic, Deduction,* p. 251). The implication here made that the proof of rules of conversion is a kind of inductive proof seems to me unwarranted.

The justification of conversion that I should myself give is that in the case of each of the four fundamental forms of proposition, its conversion (or in the case of an **O** proposition, the impossibility of converting it) is self-evident; and that we cannot go beyond this simple statement. Thus, taking an **E** proposition, I should say that it is self-evident that if one class is entirely excluded from another class, this second class is entirely excluded from the first. In the case of an **A** proposition it is clear on reflection that the statement All S is P may include one or other of the two relations of classes,—either S and P coincident, or P containing S and more besides,—but that these are the only two possible relations to which it can be applied. It is self-evident that in each of these cases some P is S; and hence the inference by conversion from an **A** proposition is shewn to be justified[1]. In the case of an **O** proposition,

---

[1] Compare section 95, where these inferences are illustrated by the aid of the Eulerian diagrams.

if we consider all the relationships of classes in which it holds good, we find that nothing is true of P in terms of S in *all* of them. Hence **O** is inconvertible[1]. I may add that I do not see that in the above reasoning we should be assisted by any explicit reference to the three laws of thought; nor that the application of the three laws of thought alone would be sufficient to give us our results.

103. Without assuming Conversion, how would you logically justify the process of Contraposition ? [C.]

---

[1] Again, compare section 95.

CHAPTER VIII.

PREDICATION AND " EXISTENCE[1]."

104. Are assumptions with regard to " existence " involved in any of the processes of immediate inference ?

As pointed out by Mr Venn (*Symbolic Logic*, pp. 127, 128), a discussion about " existence " need not in this connection involve us in any kind of metaphysical enquiry. " As to the nature of this existence, or what may really be meant by it, we have hardly any need to trouble ourselves, for almost any possible sense in which the logician can understand it will involve precisely the same difficulties and call for the same solution of them. We may leave it to any one to define the existence as he pleases, but when he has done this it will always be reasonable to enquire whether there is anything existing corresponding to the X or Y which constitute our subject and predicate. There can in fact be no fixed tests for this existence, for it will vary widely according to the nature of the subject-matter with which we are concerned in our reasonings. For instance, we may happen to be speaking of ordinary phenomenal existence, and at the time present; by the distinction

---

[1] It may perhaps be advisable for students, on a first reading, to omit this chapter.

in question is then meant nothing more and nothing deeper than what is meant by saying that there are such things as antelopes and elephants in existence, but not such things as unicorns or mastodons. If again we are referring to the sum-total of all that is conceivable, whether real or imaginary, then we should mean what is meant by saying that everything must be regarded as existent which does not involve a contradiction in terms, and nothing which does. Or if we were concerned with Wonderland and its occupants we need not go deeper down than they do who tell us that March hares exist there. In other words, the interpretation of the distinction will vary very widely in different cases, and consequently the tests by which it would have in the last resort to be verified; but it must always exist as a real distinction, and there is a sufficient identity of sense and application pervading its various significations to enable us to talk of it in common terms."

Now, several views may be taken as to what implication with regard to existence, if any, is involved in any given proposition.

(1) It may be held that every proposition implies the existence of its subject, since there is no use in giving information with regard to a non-existent subject.

(2) It may be held that although such existence is generally implied, still it is not so necessarily; and that at any rate in Formal Logic we ought to leave entirely on one side the question of the existence or the non-existence of the subjects of our propositions.

(3) The view is taken by Mr Venn that for purposes of Symbolic Logic, *universal* propositions *should not* be regarded as implying the existence of their subjects, but that *particular* propositions *should* be regarded as doing so. This view might be extended to ordinary Formal Logic.

Without at once deciding which of these views is to be preferred, we may briefly investigate the consequences which follow from them respectively so far as immediate inferences are concerned.

First, we may take the supposition that *every proposition implies the existence of its subject.* Thus, All S is P implies the existence of S, and it follows that it also implies the existence of P. No S is P implies the existence of S, and since by the law of excluded middle every S is either P or not-P, it follows that it also implies the existence of not-P.

But now if from All S is P we are to be allowed to obtain the ordinary immediate inferences,—if, for example, we may infer All not-P is not-S,—the existence of not-P and not-S are also involved. Similarly, the conversion of No S is P requires that we posit the existence of P and not-S.

On this supposition, then, we find that *propositions are not amenable to the ordinary logical operations, except on the assumption of the existence of classes corresponding not merely to the terms directly involved but also to their contradictories.*

De Morgan practically adopts this alternative. "By the *universe* (of a proposition) is meant the collection of all objects which are contemplated as objects about which assertion or denial may take place. *Let every name which belongs to the whole universe be excluded as needless:* this must be particularly remembered. Let every object which has not the name X (*of which there are always some*) be conceived as therefore marked with the name x meaning not-X" (*Syllabus*, pp. 12, 13). Compare also Jevons, *Pure Logic*, pp. 64, 65; *Studies in Deductive Logic*, p. 181.

Secondly, we may take the supposition that *no proposition logically implies the existence of its subject.* On this view, the proposition All S is P may be read, All S, *if* there is any S, or, *when* there is any S, is P; and its full implication

with regard to existence may be expressed by saying that it
denies the existence of any thing that is at the same time S
and not P. In Mr Venn's words, "*the burden of implication
of existence is shifted from the affirmative to the negative form;*
that is, it is not the existence of the subject or the predicate
(in affirmation) which is implied, but the non-existence of
any subject which does *not* possess the predicate" (*Symbolic
Logic*, p. 141). Similarly, on this view, No S is P implies
the existence neither of S nor of P, but merely denies the
existence of anything that is both S and P. Some S is P
(or is not P) may be read Some S, if there is any S, is P
(or is not P). Here we do not even negative or deny the
existence of any class absolutely; the sum total of what we
affirm with regard to existence is that *if* any S exists, then
some P (or not-P) also exists.

Now having got rid of the implication of the existence
of the subject in the case of all propositions, we might
naturally suppose that in no case in which we make an
immediate inference need we trouble ourselves with any
question of "existence" at all. On further enquiry, however,
we shall find that so far as particulars are obtained, assump-
tions with regard to existence are still involved in some
processes of immediate inference.

All S is P at any rate implies that if there is any S there
is also some P, whilst on our present view it does not re-
quire that if there is any P there is also some S. But the
converse of the given proposition,—Some P is S,—does
imply this. "If the predicate exists then also the subject
exists" must therefore be regarded as an assumption which
is involved in the conversion of an **A** proposition; similarly,
in the conversion of an **I**, and in the contraposition of an **E**
or of an **O** proposition. It follows also that in passing
from All S is P to Some not-S is not-P, we have to assume

that if there is any not-S there is also some not-P.   It
does not appear that there is any similar assumption in the
conversion of an **E** proposition ; nor do I think that there
is any in the obversion either of **A** or **E**, or in the contra-
position of **A**.   It might indeed at first sight seem that in
passing from No S is P to All S is not-P, we have to assume
that if there is any S there is also some not-P.   But, even
on our present supposition, this is necessarily implied in
the proposition No S is P itself.   If there is any S it is
by the law of excluded middle either P or not P; therefore,
given that No S is P, it follows immediately that if there
is any S there is some not-P.   It can also be shewn that
since No S is P denies the existence of anything that is
both S and P, it implies by itself that if there is any P
there is some not-S; and that since the proposition All S
is P denies the existence of anything that is both S and not-
P, it implies by itself that if there is any not-P there is some
not-S.

The given supposition then provides for the obversion
and contraposition of **A**, and for the obversion and con-
version of **E**, without any further implication with regard
to existence than is contained in the propositions them-
selves.   But the conversion or inversion of **A** involves
further assumptions, as shewn above ; and the same is true
of the contraposition or inversion of **E**, the conversion of
I and the contraposition of **O**.

Now it will be observed that in the first set of cases
we obtain by our immediate inference a *universal* propo-
sition ; in the second set a *particular* one.   We may there-
fore generalise our results as follows,—On the supposi-
tion that no proposition logically implies the existence of its
subject *we do not require to make any assumption with regard
to existence in any process of immediate inference provided that*

it yields a universal conclusion; but it is generally other-wise in cases that yield only a particular conclusion. In other words, whenever we are left with a universal con-clusion we need not be afraid that any assumption with regard to existence has been introduced unawares; but whenever we are left with a particular conclusion such an assumption may have been made, and if we find that it is so, this should be explicitly stated.

Thirdly, taking Mr Venn's view, which is the same as the preceding so far as universal propositions are concerned, but which regards particular propositions as implying the existence of their subjects, the result just obtained hardly requires to be modified. It must however be observed that on this supposition we cannot even pass from All S is P to Some S is P, except under the condition that the existence of S is granted.

105. Shew that in some processes of conversion assumptions as to the existence of classes in nature have to be made; and illustrate by examining whether any such assumptions are involved in the inference that if All S is P, therefore Some not-S is not-P.

Concerning this question, Professor Jevons remarks that it "must have been asked under some misapprehension. The inferences of Formal Logic have nothing whatever to do with real existence; that is, occurrence under the con-ditions of time and space" (*Studies in Deductive Logic*, p. 55). The question is doubtless somewhat unguarded with regard to the nature of the existence implied, but I think that in any case the discussion in the preceding section shews that it does not admit of being so summarily dis-missed[1]. Even granting that the formal logician may say

[1] What follows is to some extent a repetition of what has been

that given the proposition All S is P, it is no concern of his whether or not there are any individuals actually belonging to the classes S and P, nevertheless he must admit that the proposition at least involves that if there are any S there must be some P, while it does not involve that if there are any P there must be some S. But now convert the proposition. We obtain Some P is S, and this does involve that if there are any P there must be some S. I do not therefore see how in converting the given proposition this assumption can be avoided. Thus, from "All dragons are serpents", we may infer by conversion "Some serpents are dragons," and this proposition implies that if there are serpents there are also dragons. Similarly, in passing from All S is P to Some not-S is not-P, it must at least be assumed that if S does not constitute the entire universe of discourse, neither does P do so. If we make immediate inferences from hypothetical propositions, the necessity of a similar assumption seems still more obvious. For example, from the true statement that if Governor Musgrave's economic doctrines are correct, Mr Mill makes mistakes in his Political Economy, we can hardly without qualification infer that in some cases in which Mr Mill makes mistakes in his Political Economy, Governor Musgrave's doctrines are correct, since Mr Mill might be sometimes wrong, and nevertheless Governor Musgrave always so.

In another place (*Studies in Deductive Logic*, p. 141) Jevons remarks, " I do not see how there is in deductive logic any question about existence"; and with reference to the opposite view taken by De Morgan, he says, " This is one of the few points in which it is possible to suspect him

given in the preceding section. The view that I am here especially combating however is that Formal Logic cannot possibly have any concern with questions relating to " existence."

of unsoundness." I can however attach no meaning to Jevons's own " Criterion of Consistency " (*Studies in Deductive Logic*, p. 181) unless it has some reference to "existence." " It is assumed as a necessary law that every term must have its negative. This was called the *Law of Infinity* in my first logical essay (*Pure Logic*, p. 65; see also p. 45); but as pointed out by Mr A. J. Ellis, it is assumed by De Morgan, in his *Syllabus*, Article 16. Thence arises what I propose to call the *Criterion of Consistency*, stated as follows :— *Any two or more propositions are contradictory when, and only when, after all possible substitutions are made, they occasion the total disappearance of any term, positive or negative, from the Logical Alphabet.*" What can this mean but that although we may deny the existence of the combination *AB*, we cannot without contradiction deny the existence of *A* itself, or not-*A*, or *B*, or not-*B*? Indeed, in reference to Jevons's equational logic generally, what can negativing a combination mean but denying its existence? For example, I take the following quite at random,—" There remain four combinations, *ABC, aBC, abC*, and *abc*. But these do not stand on the same logical footing, because if we were to remove *ABC*, there would be no such thing as *A* left; and if we were to remove *abc* there would be no such thing as *c* left. Now it is the Criterion or condition of logical consistency that every separate term and its negative shall remain. Hence there must exist some things which are described by *ABC*, and other things described by *abc*" (*Studies in Deductive Logic*, p. 216).

With regard to Jevons's criterion of consistency itself, I am hardly prepared to admit it. If I am not allowed to negative *X*, why should I be allowed to negative *AB*? There is nothing to prevent *X* from being itself a complex term. In certain combinations indeed it may be convenient

to substitute X for AB, or *vice versa*. It would appear then that what is contradictory when we use a certain set of symbols may not be contradictory when we use another set of symbols. I should say that Jevons's criterion is sometimes a convenient assumption to make, but nothing more than this; and it is I think an assumption that should always be explicitly referred to when made.

106. Is a categorical proposition to be regarded as logically implying the existence of its subject?

Our answer to this question must depend to some extent on popular usage, and to some extent on logical convenience. So far as *universal* propositions are concerned, I should be inclined on both grounds to answer it in the negative.

In the first place, I do not think that in ordinary speech we always imply the existence of the subjects of our propositions. No doubt we usually regard them as existing; but as Mr Venn shews there are undoubtedly exceptions to this rule. " For instance, assertions about the *future* do not carry any such positive presumption with them, though the logician would commonly throw them into precisely the same 'All X is Y' type of categorical assertion. 'Those who pass this examination are lucky men' would certainly be tacitly supplemented by the clause 'if any such there be.' So too, in most circumstances of our ordinary life, wherever we are clearly talking of an ideal. 'Perfectly conscientious men think but little of law and rule,' has a sense without implying that there are any such men to be found[1] " (*Symbolic Logic*, pp. 130, 131). Again, a mathe-

[1] The above seems to me an answer to such a statement as the following :—" In an ordinary proposition the subject is necessarily admitted to exist, either in the real or in some imaginary world assumed

matician might assert that a rectilinear figure having a million equal sides and inscribable in a circle has a million equal angles, without intending to imply the actual existence of such a figure; or if I know that A is X, B is Y, C is Z, I may affirm that ABC is XYZ without wishing to commit myself to the view that the combination ABC does ever really occur[1]. Taking complex subjects, and limiting our conception of existence as we not unfrequently do to some particular universe, cases of this kind might be multiplied indefinitely.

But if it is granted that in ordinary thought the existence of the subject of the proposition sometimes is and sometimes is not implied, it follows that since the logician cannot discriminate between these cases, he had best content himself with leaving the question open, that is, he should regard such existence as not necessarily or logically implied.

And, further, to adopt this alternative is logically more convenient, since so far as the obtaining *universal* propositions by immediate inference is concerned, we do not on this supposition require any further assumptions with regard to existence in order that such immediate inference may be legitimate. On the other hand, if we take the other alter-

---

for the nonce......When we say *No stone is alive*, or *All men are mortal*, we presuppose the existence of stones or of men. Nobody would trouble himself about the possible properties of purely problematic men or stones" (*Mind*, 1876, pp. 290, 291). But the conclusions, "Those who pass this examination are lucky men," "Perfectly conscientious men think but little of law and rule" may certainly be worth obtaining, although in the universe to which reference is made, (and in both the cases in question this would be the actual material universe), the subjects of these propositions might be non-existent.

[1] Is it not sometimes the case that in order to disprove the existence of some combination, say AB, we establish a self-contradictory proposition of the form AB is both C and not-C?

native and regard categorical propositions as always im-
plying the existence of their subjects, we have shewn in
section 104 that we require to assume the existence not
merely of the actual terms involved in any given proposi-
tion, but also of their contradictories.

The importance of the question here raised is more
particularly manifest when we are dealing with very complex
propositions, as is shewn by Mr Venn.

We say then that logically All S is P implies only the
non-existence of anything that is both S and not-P; No S
is P implies only the non-existence of anything that is both
S and P.

The case of *particular* propositions still remains ; and
here again I am inclined to agree with the view taken by
Mr Venn in his *Symbolic Logic*, namely that such proposi-
tions should be regarded as implying the existence of their
subjects. The chief grounds for adopting this view is that
" an assertion confined to ' some ' of a class generally rests
upon observation or testimony rather than on reasoning or
imagination, and therefore almost necessarily postulates
existent data, though the nature of this observation and
consequent existence is, as already remarked, a perfectly
open question " (*Symbolic Logic*, p. 131). I doubt whether
in ordinary speech we ever predicate anything of a non-
existent subject unless we do so universally. The principal
objection to this view is perhaps the paradox which follows
from it, namely that we are not without qualification justified
in inferring from All S is P that Some S is P, (since the
latter proposition implies the existence of S, while the
former does not). It may even be said that this view
practically banishes the particular proposition from Logic
altogether. Possibly if it were so, it would be no very
serious matter. But I do not think that it is so. We have

only to be careful in using such propositions to note the assumption involved in their use. The principal value of particulars is in their relation of contradiction to universals of different quality. But their use in this respect is entirely consistent with the above. We have taken the view that the import of All S is P is to deny that there is any S that is not-P; we are now taking the view that the import of Some S is not P is to affirm that there is some S that is not-P. This clearly brings out the contradictory character of the two propositions. Similarly with **I** and **E**.

One interesting point to notice here is that if there is no implication of the existence of the subject in universal propositions we are not actually precluded from asserting together two contraries. We may say All S is P and No S is P; but this virtually is to deny the existence of S.

All S *is* P excludes

$$\left(\ S \ \middle)\middle(\ P\ \right),\quad \left(\begin{array}{c} P \\ S \end{array}\right),\quad \left(\ S\ \right)\left(\ P\ \right);$$

No S *is* P excludes

$$\left(\ S\ P\ \right),\quad \left(\begin{array}{c} S \\ P \end{array}\right).$$
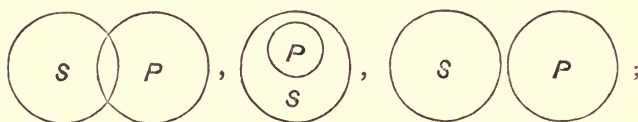
But these are all possible cases.

In other respects, this investigation if pursued might somewhat modify accepted logical doctrines ; but I feel convinced that we should be ultimately left with a consistent whole.

The truth is, as Mr Venn has remarked, that most English logicians have made no critical examination at all

of the question here raised. It may be desirable to return to it briefly in connection with the syllogism. Compare sections 273—277.

[The above view, which is taken by Mr Venn in respect to Symbolic Logic, and which I have attempted to .apply to ordinary Formal Logic, is practically identical with that somewhat recently put forward in a more paradoxical form by Professor Brentano. Compare *Mind*, 1876, pp. 289—292. "Where we say *Some man is sick*, Brentano gives as a substitute, *There is a sick man*. Instead of *No stone is alive*, he puts *There is not a live stone*. *Some man is not learned* becomes *There is an unlearned man*. Finally, *All men are mortal* is to be expressed in his system *There is not an immortal man*."]

107. Discuss the relation between the propositions All *S* is *P* and All not-*S* is *P*.

This is an interesting case to notice in connection with the discussion raised in the preceding sections.

All *S* is *P* = No *S* is not-*P* = No not-*P* is *S*.

All not-*S* is *P* = No not-*S* is not-*P* = No not-*P* is not-*S*

= All not-*P* is *S*.

The given propositions come out therefore as contraries.

(1) On the view that we ought not to enter into any discussion concerning "existence" in connection with immediate inference, we must I suppose rest content with this statement of the case. It seems however sufficiently curious to demand further investigation and explanation.

(2) On the view that propositions imply the existence of their subjects, we have shewn in section 104, that we are not justified in passing from All not-*S* is *P* to All not-*P* is *S* unless we assume the existence of not-*P*. But it will

be observed that in the case before us, the given propo-
sitions make such an assumption unjustifiable.   Since All S
is P and All not-S is P, and everything is either S or not-S
by the law of excluded middle, it follows that nothing is
not-P.

In reducing the given propositions therefore to such a
form that they appear as contraries, (and therefore as in-
consistent with each other), we assume the very thing that
taken together they really deny.

(3)   On the view that at any rate universal propositions
do not imply the existence of their subjects, we have shewn
in the preceding section, that the propositions No not-P is
S, All not-P is S, are either inconsistent or else they express
the fact that P constitutes the entire universe of discourse.
But this fact is the very thing that is given us by the propo-
sitions in their original form.

On either of the views (2) or (3), then, the result obtained
is satisfactorily accounted for and explained.

108. The nature of the logical distinction between Categorical and Hypothetical Propositions.

Are the propositions "All B is C" and "If anything is B, it is C" equivalent? or can either be inferred from the other?

Mr Venn holds that the real *differentia* of Hypothetical Propositions is "to express human doubt" (*Mind*, 1879, p. 42). I should myself prefer to express the import of Hypothetical Propositions by saying that they affirm a connection between certain events, whenever they happen or if they ever happen, whilst leaving the question entirely open whether or not they do ever happen. The doubt which they imply is rather incidental, than the fundamental or differentiating characteristic belonging to them. *Materially* indeed I think that they do sometimes imply the actual occurrence of their antecedents. Whenever the connection between the antecedent and the consequent in a hypothetical proposition can be inferred from the nature of the antecedent independently of specific experience, (and this may be the more usual case), then the actual happening of the antecedent is not in any sense in-

volved; but if our knowledge of the connection does depend on specific experience, (as it sometimes may), and could not have been otherwise obtained, then such actual happening would appear to be materially involved.   For example, the statement, "If we descend into the earth, the temperature increases at a nearly uniform rate of 1° Fahr. for every 50 feet of descent down to almost a mile," requires that actual descents into the earth should have been made, for otherwise the truth of the statement could not have been known.

It may, however, be replied that the doubt applies to the actual occurrence of the antecedent *in a given instance*. When I say "If the glass falls, it will rain," I imply doubt as to whether it actually will fall on the occasion to which I am referring. (Compare Venn, *Symbolic Logic*, pp. 331—333.) But may not this be the case also with categorical propositions?   For example, if I am in doubt whether a given plant is an orchid, I may apply the proposition "All orchids have opposite leaves" in order to resolve my doubt.   We have such a case as this whenever categorical propositions are used in the process of diagnosis, and it can hardly be said that we never do employ categorical propositions in this manner.

Still, it is clear that the hypothetical proposition does not *necessarily* imply the actual occurrence of its antecedent; and therefore, if the view is taken that the categorical proposition does necessarily imply the actual existence of its subject, (compare sections 104, 106), we have a marked distinction between the two kinds of propositions.   "If anything is B, it is C" cannot be resolved into "All B is C", since the latter implies the existence of B while the former does not.

Another view with regard to categorical propositions,

and the one for which I have expressed a preference, is that
they do not *necessarily* imply, (and therefore do not *logically*
imply), the existence of their subjects. On this view, I do
not see that we have any logical distinction between hypo-
thetical and categorical propositions, except a distinction of
form ; that is, they may be resolved into one another. We
may say indifferently " All *B* is *C*" or " If anything is *B* it
is *C*"; " If *A* is *B*, *C* is *D*" or " All cases of *A* being *B* are
cases of *C* being *D*."

Kant denies that we can reduce the hypothetical judg-
ment to the categorical form on the following ground : " In
categorical judgments nothing is problematical, but every-
thing assertative; in hypothetical it is merely the connection
between the antecedent and the consequent that is assertative.
Hence here we may combine two false judgments." This
view has I think been virtually discussed in what I have
already said. If the categorical judgment is regarded as
affirming not merely a connection between the subject and
the predicate but also the existence of the subject, then I
admit the force of the above argument, and allow that the
hypothetical judgment cannot be reduced to the categorical
form. But *if the categorical judgment is not regarded as
affirming the existence of the subject*, it (like the hypothetical
judgment) asserts no more than a *connection ;* it is no more
assertative than the hypothetical judgment, and just as
problematic. The non-existence of the subject of the cate-
gorical corresponds exactly to the falsity of the antecedent
of the hypothetical ; and if in the latter we may combine
two false judgments, in the former we may combine two
non-existent entities. I may say, If *A* is *B*, *C* is *D*, although
A is *B* is a false judgment ; but similarly I may say any
case of *A* being *B* is a case of *C* being *D*, although
the case of *A* being *B* is a non-existent case. I cannot

see that in the latter of these statements I have committed myself to anything whatever that is not contained in the former.

Hamilton also (*Logic*, I. p. 239) holds that a hypothetical judgment cannot be converted into a categorical. "The thought, *A* is through *B*, is wholly different from the thought, *A* is in *B*. The judgment,—If God is righteous, then will the wicked be punished, and the judgment,—A righteous God punishes the wicked, are very different, although the matter of thought is the same. In the former judgment, the punishment of the wicked is viewed as a consequent of the righteousness of God; whereas the latter considers it as an attribute of a righteous God. But as the consequent is regarded as something dependent from,—the attribute, on the contrary, as something inhering in, it is from two wholly different points of view that the two judgments are formed." Now it must certainly be admitted that in any given instance there are reasons why we choose the hypothetical mode of expression rather than the categorical, or *vice versa* ; but the only question that concerns us from a logical point of view is whether precisely the same meaning cannot be expressed in either form. Hamilton would appear to deny not merely that a hypothetical judgment can be converted into a categorical, but also that a categorical can be converted into a hypothetical. But, (leaving on one side the question of the existence of the subject in a categorical proposition, which has already been discussed), can any one who allows that "all orchids have opposite leaves" deny that "if this plant is an orchid it has opposite leaves"? Can any one who allows that "if there are sharpers in the company we ought not to gamble," deny that "all cases in which there are sharpers in the company are cases in which we ought not to gamble"?

If this is admitted, the logical question is to my mind disposed of[1].

No doubt hypothetical propositions will frequently look awkward when expressed in the categorical form, but in some cases logical error is more likely to be avoided if we reduce them to this form before manipulating them; and I cannot see how we lose anything, or, (on the view now taken with regard to the existential import of categorical propositions), imply anything that we should not imply, in so dealing with them. I have given examples shewing that the doctrines of opposition and immediate inference may be applied to hypotheticals. We shall find that the same is true of the doctrine of syllogism, though it may be useful to frame special rules when we are dealing with propositions expressed in this form.

109. The interpretation of Disjunctive Propositions.

There is a difference of opinion among logicians as to

[1] Mansel's view upon this question (*Aldrich*, pp. 103, 104) is not easy to understand. He admits however that "If A is B, C is D" implies that "Every case of A being B is a case of C being D." He even goes so far as to resolve "If all A is B, all A is C" into "All B is C," which is clearly erroneous. His whole treatment of hypotheticals is puzzling. For example, he says, "The judgment, 'If A is B, C is D,' asserts the existence of a consequence necessitated by laws other than those of thought, and consequently out of the province of Logic" (*Aldrich*, p. 236; *Prolegomena Logica*, p. 230). But similarly a categorical proposition may assert a connection not necessitated by laws of thought; and I do not see that we have here any reason for subjecting hypothetical propositions to a peculiar treatment. I am inclined to think that what makes Mansel's discussion of hypothetical propositions so difficult is that he attempts to apply to them the strict conceptualist view of Logic, which it is impossible to apply consistently throughout without divesting Logic of all content whatsoever.

whether the alternatives in a disjunctive proposition should
be regarded as mutually exclusive. For example, in the
proposition A is either B or C, there is not general agree-
ment as to whether it is logically implied that A cannot be
both B and C[1].

There are at least two questions involved which should
be distinguished.

(1) In ordinary speech do we intend that the alter-
natives in a disjunctive proposition should be necessarily
understood as excluding one another? A very few instances
will I think enable us to answer this question in the
negative. "Take, for instance, the proposition—' A peer
is either a duke, or a marquis, or an earl, or a viscount, or
a baron'...Yet many peers do possess two or more titles,
and the Prince of Wales is Duke of Cornwall, Earl of
Chester, Baron Renfrew, &c....In the sentence—' Repent-
ance is not a single act, but a habit or virtue,' it cannot be
implied that a virtue is not a habit....Milton has the ex-
pression in one of his Sonnets—' Unstain'd by gold or fee,'
where it is obvious that if the fee is not always gold, the
gold is a fee or bribe. Tennyson has the expression
'wreath or anadem.' Most readers would be quite un-
certain whether a wreath may be an anadem, or an anadem
a wreath, or whether they are quite distinct or quite the
same" (Jevons, *Pure Logic*, pp. 76, 77).

(2) But this does not absolutely settle the question.
It may be said:—Granted that in common speech the
alternatives of a disjunction may or may not be mutually
exclusive, still in Logic we should be more precise, and

[1] Whately, Mansel, Mill, and Jevons would answer this question
in the negative; Kant, Hamilton, Thomson, Boole, Bain, and Fowler
in the affirmative.

the statement "A is either B or C" (where it may be both)
should be written "A is either B or C or both."

This is a question of interpretation or method, and I do
not apprehend that any burning principle is involved in the
answer that we may give. For my own part I do not
find any reason for diverging from the usage of everyday
language. On the other hand, I think that if Logic is
to be of practical utility, the less logical forms diverge
from those of ordinary speech the better. And further, it
conduces to clearness if we make a logical proposition
express as little as possible. "A is either B or C, it can-
not be both" is best given as two distinct propositions[1].

___

[1] A view strongly opposed to that adopted in the text is taken in a
recently published work on the *Principles of Logic* by Mr Bradley of
Merton College, Oxford. His argument is as follows:—" The com-
monest way of regarding disjunction is to take it as a combination of
hypotheses. This view in itself is somewhat superficial, and it is
possible even to state it incorrectly. ' Either A is B or C is D' means,
we are told, that if A is not B then C is D, and if C is not D then
A is B. But a moment's reflection shews us that here two cases are
omitted. Supposing, in the one case, that A *is* B, and supposing,
in the other, that C *is* D, are we able in these cases to say nothing at
all? Our ' either—or' can certainly assure us that, if A is B, C—D
must be false, and that, if C is D, then A—B is false. We have
not exhausted the disjunctive statement, until we have provided for
four possibilities, B and not-B, C and not-C" (*Principles of Logic*,
p. 121). The question raised is really one of interpretation, as I have
indicated above; but this is what Mr Bradley will not admit. In my
view, it is open to a logician to choose either of the two ways of
interpreting a disjunctive proposition, provided that he makes it quite
clear which he has selected ; but I can see no good in dogmatising as
in the following passage,—" Our slovenly habits of expression and
thought are no real evidence against the exclusive character of dis-
junction. 'A is b or c' does strictly exclude 'A is both b and c.'
When a speaker asserts that a given person is a fool or a rogue, he
may not *mean* to deny that he is both. But, having no interest in

Professor Fowler indicates this view in his statement that
" it is the object of Logic not to state our thoughts in
a condensed form but to analyse them into their simplest
elements" (*Deductive Logic*, p. 32); though he does not
apply it to the case before us.

Mansel arguing in favour of the view that I have taken
remarks,—" But let us grant for a moment the opposite
view, and allow that the proposition, ' All *C* is either *A* or
B,' implies, as a condition of its truth, ' No *C* can be both.'
Thus viewed, it is in reality a complex proposition, contain-
ing two distinct assertions, each of which may be the ground
of two distinct processes of reasoning, governed by two
opposite laws.   Surely it is essential to all clear thinking,
that the two should be separated from each other, and not
confounded under one form by assuming the Law of Ex-
cluded Middle to be, what it is not, a complex of those
of Identity and Contradiction" (*Prolegomena Logica*, p. 238).

Of course if the alternatives are logical contradictories
they are logically exclusive, but otherwise in the treatment
of disjunctive propositions in the following pages I do not
regard them as being so.   If in any case they happen to be
materially incompatible, this must be separately stated.

110.   From the statement that blood-vessels are
either veins or arteries, does it follow logically that a
blood-vessel, if it be a vein, is not an artery ?   Give
your reasons.   [L.]

shewing that he is both, being perfectly satisfied provided he is one,
either *b* or *c*, the speaker has not the possibility *bc* in his mind.   Ig-
noring it as irrelevant, he argues as if it did not exist.   And thus he
may practically be right in what he says, though formally his statement
is downright false : for he has excluded the alternative *bc*" (p. 124).

111. Put, if you can, the whole meaning of a disjunctive proposition (such as, Either A is B or C is D)
in the form of a single and simple Hypothetical, and
prove your expression to be sufficient.   [R.]

Adopting the view that in a disjunctive proposition the
alternatives are not to be regarded as necessarily excluding
one another, such a disjunctive proposition as the above is
primarily reducible to two hypotheticals, namely, If A is not
B, C is D, and If C is not D, A is B. But each of these
is the contrapositive of the other, and may therefore be inferred from it. Hence the full meaning of the disjunctive
is expressed by means of *either* of these hypotheticals[1].

Professor Croom Robertson called attention to this
point in *Mind*, 1877, p. 266,—"The other form of proposition ranged by logicians with the Hypothetical, namely the
Disjunctive, may be shewn to be as simple as the pure
Hypothetical being in fact a special case of it. The common view is that it involves at least two hypothetical propositions, or, as some say, even four. Thus ' Either A is B
or C is D' is resolved by some into the four hypotheticals—

---

[1] Mr Bradley (*Principles of Logic*, p. 121), lays it down that
"disjunctive judgments cannot really be reduced to hypotheticals"
at all; but I hardly care to disagree with him since he admits all that
I should contend for. He distinctly resolves "A is b or c" into
hypotheticals (p. 130); but, he adds, although the meaning of disjunctives can thus "be given hypothetically; we must not go on to
argue from this that they *are* hypothetical" (p. 121). They "declare
a fact without any supposition " (p. 122). But so does the hypothetical
itself, namely, the connection between the antecedent and the consequent. Further, "A *combination* of hypotheticals surely does not lie
in the hypotheticals themselves" (p. 122). Undoubtedly, by means of
a combination of hypotheticals, we may make a most categorical statement; *e.g.*, If A is B, C is D; and if A is not B, C is D.

If A is B, C is not D (1),

If A is not B, C is D (2),

If C is D, A is not B (3),

If C is not D, A is B (4),

—but the first and third of these are rejected by others, and with reason, because they are in fact implied only when the alternatives are logical opposites. The remaining propositions (2) and (4) are, however, the logical contrapositives of one another; and this amounts to saying that either of them *by itself* is a full and adequate expression of the original disjunctive."

PART III.

SYLLOGISMS.

CHAPTER I.

THE RULES OF THE SYLLOGISM.

112. The Terms of the Syllogism.

A reasoning consisting of three categorical propositions (of which one is the conclusion), and containing three and only three terms, is called a Categorical Syllogism.

Every categorical syllogism then contains three and only three terms, of which two appear in the conclusion and also in one or other of the premisses, and one in the premisses only. That which appears as the predicate of the conclusion, and in one of the premisses, is called the *major term;* that which appears as the subject of the conclusion, and in one of the premisses, is called the *minor term;* and that which appears in both the premisses, but not in the conclusion, (being that term by their relations to which the mutual relation of the two other terms is determined), is called the *middle term.*

Thus, in the syllogism,—

> All *M* is *P*,
> All *S* is *M*,
> therefore, All *S* is *P*;

P is the major term, *S* is the minor term, and *M* is the middle term.

[These respective designations of the terms of a syllogism resulted from such a syllogism as,—

> All *M* is *P*,
> All *S* is *M*,
> therefore, All *S* is *P*,

being taken as the type of syllogism. With the exception of the somewhat rare case in which the terms of a proposition are coextensive, such a syllogism as the above may be represented by the following diagram. Here clearly the
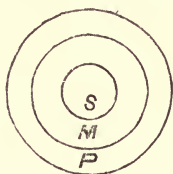
major term is the largest in extent, and the minor the smallest, while the middle occupies an intermediate position. But we have no guarantee that the same relation between the terms of a syllogism will hold, when one of the premisses is a negative or a particular proposition; *e.g.*, the following syllogism,—

> No *M* is *P*,
> All *S* is *M*,
> therefore, No *S* is *P*,

gives as one case
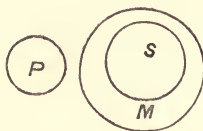
where the major term may be the smallest in extent, and the middle the largest.

Again, the following syllogism,—

No M is P,

Some S is M,

therefore, Some S is not P,

gives as one case
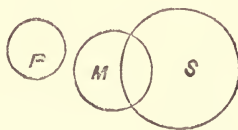
where the major term may be the smallest in extent and the minor the largest.

With regard to the middle term, however, we may note that although it is not always a middle term in extent, it is always a middle term in the sense that by its means the two other terms are connected, and their mutual relation determined.]

113. The Propositions of the Syllogism.

Every categorical syllogism consists of three propositions. Of these one is the *conclusion*. The premisses are called the *major premiss* and the *minor premiss* according as they contain the major term or the minor term respectively.

Thus, All *M* is *P*, (major premiss),
　　　　All *S* is *M*, (minor premiss),
therefore, All *S* is *P*, (conclusion).

It is usual, (as in the above syllogism), to state the major premiss first and the conclusion last.

114. The Rules of the Syllogism; and the Deduction of the Corollaries.

The rules of the Syllogism as usually stated are as follows :—

(1) *Every syllogism contains three and only three terms.*

(2) *Every syllogism consists of three and only three propositions.*

It may be observed that these are not so much rules, as a general description of the nature of the syllogism. A reasoning which does not fulfil these conditions may be formally valid, but we should not call it a syllogism[1]. The four following rules are really rules in the sense that if, when we have got the reasoning into the form of a syllogism, they are not fulfilled, then the reasoning is invalid.

(3) *No one of the three terms of the syllogism must be used ambiguously ; and the middle term must be distributed once at least in the premisses.*

This rule is frequently given in the form: "The middle term must be distributed once at least, and must not be ambiguous," (*e.g.*, in Jevons, *Elementary Lessons*, p. 127).

---

[1] For example, *B* is greater than *C*,

　　　　A is greater than *B*,

　　therefore, *A* is greater than *C*.

Here there are four terms, since the predicate of the second premiss is "greater than *B*," and this is not the same as the subject of the first premiss "*B*."

But it is obvious that we must guard against ambiguous major and ambiguous minor as well as against ambiguous middle.

If the middle term is distributed in neither of the premisses, the syllogism is said to be subject to the fallacy of *undistributed middle.*

(4) *No term must be distributed in the conclusion which was not distributed in one of the premisses.*

The breach of this rule is called *illicit process of the major*, or *illicit process of the minor*, as the case may be ; or, more briefly, *illicit major* or *illicit minor.*

(5) *From two negative premisses nothing can be inferred.*

(6) *If one premiss is negative, the conclusion must be negative; and to prove a negative conclusion, one of the premisses must be negative.*

From these rules, three corollaries may be deduced:—

(i) *From two particular premisses nothing can be inferred.*

Two particular premisses must be either

      (α) both negative,

or   (β) both affirmative,

or   (γ) one negative and one affirmative.

But in case (α), no conclusion follows by rule 5.

In case (β), since no term can be distributed in two particular affirmative propositions, the middle term cannot be distributed, and therefore no conclusion follows by rule 3.

In case (γ), if we can have a conclusion it must be negative (rule 6). The major term therefore will be distributed in the conclusion ; and hence we must have two terms distributed in the premisses, namely, the middle and the major (rules 3, 4). But a particular negative proposition and a

particular affirmative proposition between them distribute only one term. Therefore, no conclusion can be obtained.

[De Morgan (*Formal Logic*, p. 14) proves this corollary as follows :—"Since both premisses are particular in form, the middle term can only enter one of them universally by being the predicate of a negative proposition; consequently the other premiss must be affirmative, and, being particular, neither of its terms is universal. Consequently both the terms as to which the conclusion is to be drawn enter partially, and the conclusion can only be a particular *affirmative* proposition. But if one of the premisses be negative, the conclusion must be *negative*. This contradiction shews that the supposition of particular premisses producing a legitimate result is inadmissible."]

(ii) *If one premiss is particular, so must be the conclusion*[1].

We must have either

(α) two negative premisses, but this case is rejected by rule 5 ;

or (β) two affirmative premisses;

or (γ) one affirmative and one negative.

In case (β) the premisses, being both affirmative and one of them particular, can distribute but one term between them. This must be the middle term by rule 3. The minor term is therefore undistributed in the premisses, and the conclusion must be particular by rule 4.

In case (γ) the premisses will between them distribute two and only two terms. These must be the middle by

---

[1] This and the sixth rule are sometimes combined into the one rule, *Conclusio sequitur partem deteriorem*,—*i.e.*, the conclusion follows the worse or weaker premiss both in quality and in quantity; a negative being considered weaker than an affirmative, and a particular than a universal.

rule 3, and the major by rule 4, (since we have a negative premiss, necessitating a negative conclusion by rule 6, and therefore the distribution of the major term in the conclusion). Again, therefore, the minor cannot be distributed in the premisses, and the conclusion must be particular by rule 4.

[De Morgan (*Formal Logic*, p. 14) gives the following very ingenious proof of this corollary:—" If two propositions *P* and *Q*, together prove a third, *R*, it is plain that *P* and the denial of *R*, prove the denial of *Q*. For *P* and *Q* cannot be true together without *R*. Now if possible, let *P* (a particular) and *Q* (a universal) prove *R* (a universal). Then *P* (particular) and the denial of *R* (particular) prove the denial of *Q*. But two particulars can prove nothing."]

(iii) *From a particular major and a negative minor nothing can be inferred.*

Since the minor premiss is given negative, the major premiss must by rule 5 be affirmative. But it is also particular, and it therefore follows that the major term cannot be distributed in it. Hence, by rule 4, it must be undistributed in the conclusion, *i.e.*, the conclusion must be *affirmative*. But also by rule 6, since we have a negative premiss, it must be *negative*. This contradiction establishes the corollary that under the supposed circumstances no conclusion is possible.

115. Shew by aid of the syllogistic rules that the premisses of a syllogism must contain one more distributed term than the conclusion; also, that there is always the same number of distributed terms in the predicates of the premisses taken together as in the predicate of the conclusion. Hence deduce

the three corollaries. [Cf. Monck, *Introduction to Logic*, pp. 40, 41.]

116. "When one of the premisses is Particular, the conclusion must be Particular. The transgression of this rule is a symptom of illicit process of the minor." Spalding, *Logic*, p. 209. Is it the case that we cannot infer a universal conclusion from a particular premiss without committing the fallacy of illicit minor?

117. Illustrate De Morgan's statement that any case which falls under the rule that "from premisses both negative no conclusion can be inferred" may be reduced to a breach of one of the preceding rules.

De Morgan (*Formal Logic*, p. 13) takes two universal negative premisses E, E. In whatever figure they are, they can be reduced by conversion to,—

No P is M,
No S is M.

Then by obversion they become, (without losing any of their force),—

All P is not-M,
All S is not-M;

and we have undistributed middle. Hence rule 5 is exhibited as a corollary from rule 3.

An objection may perhaps be taken to the above on the ground that the premisses might also be reduced to,—

All M is not-P,
All M is not-S;

where the middle term is distributed in both premisses. Here however it is to be noted that we have no longer a middle

term *connecting S and P* at all. We shall return subsequently
to this method of dealing with two negative premisses.

The case in which one of the premisses is particular is
dealt with by De Morgan (*Formal Logic*, p. 14) as follows:—
"Again, No *Y* is *X*, Some *Y*s are not *Z*s, may be converted
into

> Every *X* is (a thing which is not *Y*),
> Some (things which are not *Z*s) are *Y*s,

in which there is no middle term."

This is not quite satisfactory, since we may often exhibit
a valid syllogism in such a form that there appear to be
four terms; *e.g.*, I might say, "All *M* is *P*, All *S* is *M*, may
be converted into

> All *M* is *P*,
> No *S* is not-*M*,

in which there is no middle term."

The case in question may however be disposed of by
saying that if we can infer nothing from two universal
negative premisses, *a fortiori* we cannot from two negative
premisses, one of which is particular.

118. The rule that "if one premiss is negative,
the conclusion must be negative," may be established
as a corollary from the rule that "from two negative
premisses nothing can be inferred."

The following has been suggested to me by De Morgan's
deduction of corollary ii., (cf. section 114):—If two pro-
positions *P* and *Q* together prove a third *R*, it is plain that
P and the denial of *R* prove the denial of *Q*. For *P* and *Q*
cannot be true together without *R*. Now if possible let *P*
(a negative) and *Q* (an affirmative) prove *R* (an affirmative).
Then *P* (a negative) and the denial of *R* (a negative) prove
the denial of *Q*. But two negatives prove nothing.

119. Simplification of the Rules of the Syllogism.

It would now seem as if the six rules of the syllogism might be simplified. Rules 1 and 2 may be treated as a description of the syllogism rather than as rules for its validity. The part of rule 3 relating to ambiguity may be regarded as contained in the proviso that there shall be only three terms, (*i.e.*, if one of the terms is ambiguous, we have not really a syllogism according to our definition of syllogism). Rule 5 has been exhibited in section 117 as a corollary from rule 3; and the first part of rule 6 has been shewn in section 118 to be a corollary from rule 5. We are left then with only three independent rules,—

(*α*) The middle term must be distributed once at least in the premisses;

(*β*) No term must be distributed in the conclusion unless it has been distributed in the premisses;

(*γ*) A negative conclusion cannot be inferred from two affirmative premisses.

120. In reference to the syllogism, it has been urged that the old rule that negative premisses yield no conclusion does not hold true universally, as in the example, Whatever is not metallic is not capable of powerful magnetic influence, carbon is not metallic, therefore, carbon is not capable of powerful magnetic influence. Examine this criticism. [C.]

Professor Jevons gives this case in his *Principles of Science* (1st edition, vol. I., p. 76; 2nd edition, p. 63), and he states that "the syllogistic rule is actually falsified in its bare and general statement."

Professor Croom Robertson has however conclusively shewn (in *Mind*, 1876, p. 219, *note*) that this apparent ex-

ception is no real exception[1]. "There are *four* terms in the
example, and thus no syllogism, if the premisses are taken
as negative propositions; while the minor premiss is an *affir-
mative* proposition, if the terms are made of the requisite
number three."

Mr Bradley (*Principles of Logic*, p. 254) returns to the
position taken by Professor Jevons. In reference to the
example given in the above question, he says, "This argu-
ment no doubt has *quaternio terminorum* and is vicious
technically, but the fact remains that from two denials you
somehow *have* proved a further denial. '*A* is not *B*, what
is not *B* is not *C*, therefore *A* is not *C*'; the premisses are
surely negative to start with, and it appears pedantic either
to urge on one side that '*A* is not-*B*' is simply positive, or
on the other that *B* and not-*B* afford no junction. If from
negative premisses I can get my conclusion, it seems idle to
object that I have first transformed one premiss ; for that
objection does not shew that the premisses are not negative,
and it does not shew that I have failed to get my con-
clusion."

This is somewhat beside the mark ; and if the points
on both sides are clearly stated there appears no room for
further controversy. On the one hand, it is implicitly
admitted both by Professor Jevons (*Studies in Deductive
Logic*, p. 89), and by Mr Bradley, that two negative
premisses invalidate a *syllogism*, *i.e.*, understanding by a
syllogism a mediate reasoning containing three and only
three terms. On the other hand, everyone would allow that
from two propositions which may both be regarded as

---

[1] Mr Venn, also, (in the *Academy*, Oct. 3, 1874),—"The reply
clearly is, that if 'not metallic' is to be regarded as the predicate of the
minor, then the minor is affirmative; if 'metallic' is predicate, then
there are four terms."

negative, a conclusion may sometimes be obtained; for example, the propositions which constitute the premisses of a syllogism in *Barbara*[1] may be written in a negative form, thus, No *M* is not-*P*, No *S* is not-*M*, and no doubt the conclusion—All *S* is *P*—still follows. We must not, however, attach undue importance to the distinction between positive and negative propositions. By means of the process of Obversion, the logician may at will regard any given proposition as either positive or negative.

[A similar case to that given in the question is dealt with in the *Port Royal Logic* (Professor Baynes's translation, p. 211) as follows :—

"There are many reasonings, of which all the propositions appear negative, and which are, nevertheless, very good, because there is in them one which is negative only in appearance, and in reality affirmative, as we have already shewn, and as we may still further see by this example :

That which has no parts cannot perish by the dissolution of its parts ;

The soul has no parts ;

Therefore, the soul cannot perish by the dissolution of its parts.

There are several who advance such syllogisms to shew that we have no right to maintain unconditionally this axiom of logic, *Nothing can be inferred from pure negatives ;* but they have not observed that, in sense, the minor of this and such other syllogisms is affirmative, since the middle, which is the subject of the major, is in it the attribute. Now the subject of the major is not that which has parts, but

---

[1]   All *M* is *P*,
            All *S* is *M*,
      therefore, All *S* is *P*.   Cf. section 158.

that which has not parts, and thus the sense of the minor is, *The soul is a thing without parts*, which is a proposition affirmative of a negative attribute."]

121. By what means can we obtain a conclusion from the two negative premisses,—

<div style="text-align:center">

No M is P,
No M is S?

</div>

By obverting the premisses, we have—,

<div style="text-align:center">

All M is not-P,
All M is not-S,
therefore, Some not-S is not-P[1].

</div>

122. Take an apparent syllogism subject to the fallacy of negative premisses, and enquire whether you can correct the reasoning by converting one or both of the premisses into the affirmative form. [Jevons, *Studies in Deductive Logic*, p. 84.]

Both in the *Studies* and in the *Principles of Science* (Vol. I., p. 75), Professor Jevons appears to answer this question in the negative. It is certainly not put in an unexceptionable form, but apparently reference is made to the case given in the preceding section.

<div style="text-align:center">

No A is B,
No A is C,

</div>

may be transformed into,—

<div style="text-align:center">

All A is not-B,
All A is not-C;

</div>

---

[1] But this does not invalidate the *syllogistic* rule that from two negative premisses nothing can be inferred, since so long as both the premisses remain negative we have more than three terms and therefore not a *syllogism* at all.

yielding a conclusion,—

<div align="center">Some not-C is not-B.</div>

[In Jevons's system, this would become,—

$$A = Ab,$$
$$A = Ac;$$

yielding a conclusion,—

$$Ab = Ac.$$

(Cf. *Principles of Science*, vol. I., p. 71; 2nd ed., p. 59).]

123. Given

    (i)   All P is M,

    (ii) All S is M,

    (iii) M does not constitute the entire universe of discourse. What conclusion can we infer?

Exhibit the reasoning in the form of an Aristotelian syllogism.

Is the third premiss necessary in order that the conclusion may be obtained? Make any comments that occur to you in connection with this point.

From (i) we can obtain by immediate inference, All not-M is not-P, and from (ii) All not-M is not-S; and these premisses yield the conclusion,—

<div align="center">Some not-S is not-P.</div>

The reasoning is here exhibited in the form of an Aristotelian syllogism.

Or, we might reason as follows:—Since S and P are both entirely included in M, there must be outside M some not-S and some not-P that are coincident; and this is the same conclusion as before.

Now in the latter form of the reasoning it would seem that we have assumed that there is *some* not-M, *i.e.*, that M

does not constitute the entire universe of discourse. But the necessity of this assumption was not apparent in our first method of treatment, according to which by a simple process of immediate inference we obtained a perfectly valid syllogism [1].

The truth appears to be that here at any rate we have an illustration of De Morgan's view (*Formal Logic*, p. 112) that in all syllogisms the existence of the middle term is a *datum*. From the premisses All M is P, All M is S, we cannot obtain the conclusion Some S is P without implicitly assuming the existence of M. Take as an example,—All witches ride through the air on broomsticks; All witches are old women; therefore, Some old women ride through the air on broomsticks. This point is further discussed in sections 273—277.

We may note that the reasoning,—

All P is M,

All S is M,

therefore, Some not-S is not-P,

does not invalidate the *syllogistic* rule that the middle term must be distributed once at least in the premisses, since as it stands it contains more than three terms and is therefore not a syllogism.

124. Examine the following assertion : " In no way can a syllogism with two singular premisses be viewed as a genuine syllogistic or deductive inference."

[W.]

This assertion is made by Professor Bain, and he illustrates it (*Logic, Deduction*, p. 159) by reference to the following syllogism :

[1] Compare, however, section 104.

Socrates fought at Delium,
Socrates was the master of Plato,
therefore, The master of Plato fought at Delium.

But "the proposition 'Socrates was the master of Plato and fought at Delium', compounded out of the two premisses is nothing more than a grammatical abbreviation "; and the step hence to the conclusion is a mere omission of something that had previously been said. "Now, we never consider that we have made a real inference, a step in advance, when we repeat *less* than we are entitled to say, or drop from a complex statement some portion not desired at the moment. Such an operation keeps strictly within the domain of Equivalence or Immediate Inference. In no way, therefore, can a syllogism with two singular premisses be viewed as a genuine syllogistic or deductive inference."

The above leads up to some very interesting considerations, but it proves too much. In the following syllogisms the premisses may be similarly compounded together,—

all men are mortal, } all men are mortal and rational;
all men are rational,
therefore, some rational beings are mortal.

all men are mortal, } all men including kings are mortal;
all kings are men,
therefore, all kings are mortal[1].

---

[1] With the above, compare the following syllogism, having two singular premisses :—

The Lord Chancellor receives a higher salary than the Prime Minister,

Lord Selborne is the Lord Chancellor,

therefore, Lord Selborne receives a higher salary than the Prime Minister.

The premisses here would similarly, I suppose, be compounded by Professor Bain into "The Lord Chancellor, Lord Selborne, receives a higher salary than the Prime Minister."

Do not Bain's criticisms apply to these syllogisms as much as to the syllogism with two singular premises? The method of treatment adopted is indeed particularly applicable to syllogisms in which the middle term is subject in both premises [1]; but in any case it is true that the conclusion of a syllogism contains a part of, and only a part of, the information contained in the two premises taken together. Also, we may always combine the two premises in a single statement; and thus we may always get Bain's result. In other words, in the conclusion of every syllogism "we repeat less than we are entitled to say," or, if we care to put it so, "drop from a complex statement some portion not desired at the moment."

It may be worth while here to refer to the charge of incompleteness which Professor Jevons (*Principles of Science*, I. p. 71) has brought against the ordinary syllogistic conclusion. "Potassium floats on water, Potassium is a metal," yield, according to him, the conclusion, "Potassium metal is potassium floating on water." But "Aristotle would have inferred that some metals float on water. Hence Aristotle's conclusion simply *leaves out some of the information afforded in the premises;* it even leaves us open to interpret the *some metals* in a wider sense than we are warranted in doing."

In reply to this it may be remarked : first, that the Aristotelian conclusion does not profess to sum up the whole of the information contained in the premises of the syllogism ; secondly, that *some* in Logic means merely "not none", "one at least". The conclusion of the above syllogism might perhaps better be written "some metal floats on water," or "some metal or metals, &c." Compare Mr Venn [2],

---

[1] *i.e.*, to syllogisms in Figure 3. Cf. section 143.

[2] "Surely, as the old expression 'discursive thought' implies, we

in the *Academy*, Oct. 3, 1874; also, Professor Croom Robertson in *Mind*, 1876, p. 219.

125. How far does the conclusion of an Aristotelian syllogism fall short of giving all the information contained in the premisses? [Jevons, *Studies*, p. 215.]

126. The connection between the *Dictum de omni et nullo* and the ordinary rules of syllogism.

The *Dictum de omni et nullo* was given by Aristotle as the axiom on which all syllogistic inference is based. It applies directly, however, to those syllogisms only in which the major term is predicate in the major premiss, and the minor term subject in the minor premiss, (*i.e.*, to what are called syllogisms in Figure 1). The rules of syllogism, on the other hand, apply independently of the position of the terms in the premisses. Nevertheless, it is interesting to trace the connection between them. We shall find all the rules implicitly contained in the *Dictum*, but some of them in a less general form, in consequence of the distinction pointed out above.

The *Dictum* may be stated as follows :—"Whatever is predicated, whether affirmatively or negatively, of a term distributed may be predicated in like manner of everything contained under it."

designedly pass on from premisses to conclusion, and then drop the premisses from sight. If we want to keep them in sight we can perfectly well retain them as premisses; if not, if all that we want is the final fact, it is no use to burden our minds or paper with premisses as well as conclusion. All reasoning is derived from data which under conceivable circumstances might be useful again, but which we are satisfied to recover when we want them."

(1) The *Dictum* provides for three and only three terms; namely, (i) a certain term which must be distributed, (ii) something predicated of this term, (iii) something contained under it. These terms are respectively the middle, major, and minor. We may consider the rule relating to the ambiguity of terms also contained here, since if any term is ambiguous we have practically more than three terms.

(2) The *Dictum* provides for three and only three propositions; namely, (i) a proposition predicating something of a term distributed, (ii) a proposition declaring something to be contained under this term, (iii) a proposition making the original predication of the contained term. These propositions constitute respectively the major premiss, the minor premiss, and the conclusion of the syllogism.

(3) The *Dictum* prescribes not merely that the middle term shall be distributed once at least in the premisses, but more explicitly that it shall be distributed in the major premiss,—"Whatever is predicated of a term *distributed*." [This is really another form of what we shall find to be a special rule of Figure 1, namely that the major premiss must be universal. Cf. section 144.]

(4) The proposition declaring that something is contained under the term distributed must necessarily be an affirmative proposition. The *Dictum* provides therefore that the premisses shall not be both negative. [It really provides that the *minor* premiss shall be affirmative, which again is one of the special rules of Figure 1.]

(5) The words "in like manner" clearly provide against a breach of rule 6, namely that if one premiss is negative, the conclusion must be negative, and *vice versa*.

(6) Illicit process of the major is provided against indirectly. We can commit this fallacy only if we have a nega-

tive conclusion, but the words "in like manner" declare
that if we have a negative conclusion, we must have a nega-
tive major premiss, and since in any syllogism to which the
Dictum directly applies, the major term is predicate of this
premiss, it likewise will be distributed.

Illicit process of the minor is simply provided against
inasmuch as we are warranted to make our predication in
the conclusion only of what has been shewn in the minor
premiss to be contained under the middle term.

127. Can the Syllogism be based exclusively on
the laws of Identity, Contradiction and Excluded
Middle ?

Mansel answers this question in the affirmative and main-
tains (*Prolegomena Logica*, p. 222) that "the Principle of
Identity is immediately applicable to affirmative moods in
any figure, and the Principle of Contradiction to negatives."
In order to shew this, he commences by quantifying the
predicate (cf. section 217), and taking as an example the
syllogism,—

<div style="text-align:center">

All *M* is some *P*,

All *S* is some *M*,

therefore, All *S* is some *P*,

</div>

he reads it thus,— "the minor term all *S* is identical with a
part of *M*, and consequently with a part of that which is
given as identical with all *M*, namely some *P*." He then
takes the syllogism,—

<div style="text-align:center">

All *M* is some *P*,

Some *S* is some *M*,

therefore, Some *S* is some *P*,

</div>

and, treating it similarly, finds that "the principle immedi-
ately applicable to both is the axiom, that what is given as
identical with the whole or a part of any concept, must be

identical with the whole or a part of that which is identical with the same concept." Passing by the inaccuracy of speaking of the *concepts* as being identical[1], I cannot see that the above axiom is the same as the Principle of Identity, "Every *A* is *A*." The syllogism is something more than mere *subaltern* inference; it involves a passage of thought through a *middle term;* and it is just this that the Law of Identity as expressed in the formula "Every *A* is *A*" appears to me unable to provide for.

This law may tell us that if all *M* is *P*, then some *M* is *P*; but does it tell us that if all *M* is *P*, therefore *S* is *P*, *because* it is *M*? The *Dictum de omni et nullo* clearly enunciates the principle involved in syllogistic reasoning; the Law of Identity, if it does so at all, does so less satisfactorily. Or rather I would say that if the Law of Identity is to cover this principle, then it is inadequately expressed in the formula Every *A* is *A*[2]. Similar remarks apply to the attempt to bring syllogisms with negative conclusions under the Principle of Contradiction, "No *A* is not-*A*."

---

[1] It is really the extension of the one concept that is identical with the whole or a part of the extension of the other; and although the comprehension of a concept is practically the concept itself, it is clear that the same is not true of its extension. It has always seemed to me rather curious that the doctrine of the Quantification of the Predicate should have been introduced by writers like Hamilton and Mansel, who lay so much stress on *concepts*.

[2] I should say the same in reference to Mansel's remark (*Prolegomena Logica*, p. 103), that the Axiom "things that are equal to the same are equal to one another" is only another statement of the Principle of Identity.

CHAPTER II.

SIMPLE EXERCISES ON THE SYLLOGISM.

128. Explain what is meant by a *Syllogism;* and put the following argument into syllogistic form :— "We have no right to treat heat as a substance, for it may be transformed into something which is not heat, and is certainly not a substance at all, namely, mechanical work." [N.]

129. Put the following argument into syllogistic form :— How can any one maintain that pain is always an evil, who admits that remorse involves pain, and yet may sometimes be a real good? [V.]

130. It has been pointed out by Ohm that reasoning to the following effect occurs in some works on mathematics :—" A magnitude required for the solution of a problem must satisfy a particular equation, and as the magnitude x satisfies this equation, it is therefore the magnitude required."

Examine the logical validity of this argument. [C.]

131. If P is a mark of the presence of Q, and R of that of S, and if P and R are never found together,

am I right in inferring that Q and S sometimes exist
separately?   [v.]

The premisses may be stated,—

<div style="text-align:center">

All P is Q,
All R is S,
No P is R;

</div>

and in order to establish the desired conclusion we must
be able to infer at least one of the following,—

<div style="text-align:center">

Some Q is not S,
Some S is not Q.

</div>

But neither of these propositions can be inferred, since
they distribute respectively S and Q, whilst neither of these
terms is distributed in the given premisses. The question
is therefore to be answered in the negative.

132. If it is false that the attribute B is ever
found coexisting with A, and not less false that the
attribute C is sometimes found absent from A, can
you assert anything about B in terms of C?   [c.]

133. Enumerate the cases in which no valid con-
clusion can be drawn from two premisses.

134. Shew that

(i)   If both premisses of a syllogism are affirma-
tive, and one but only one of them universal, they
will between them distribute only one term;

(ii)   If both premisses are affirmative and both
universal, they will between them distribute two
terms;

(iii)   If one but only one premiss is negative,

and one but only one premiss universal, they will between them distribute two terms;

(iv) If one but only one premiss is negative, and both premisses are universal, they will between them distribute three terms.

135. Ascertain how many distributed terms there may be in the premisses of a syllogism more than in the conclusion.   [L.]

136. Prove that, when the minor term is predicate in its premiss, the conclusion cannot be *A*.   [L.]

137. If the major term of a syllogism be the predicate of the major premiss, what do we know about the minor premiss?   [L.]

138. How much can you tell about a valid syllogism if you know,—

(1) that only the middle term is distributed;

(2) that only the middle and minor terms are distributed;

(3) that all three terms are distributed?   [W.]

139. If it be known concerning a syllogism in the Aristotelian system that the middle term is distributed in both premisses, what can we infer as to the conclusion?   [C.]

If both premisses are affirmative, they can between them distribute only two terms; but by hypothesis the middle term is distributed twice in the premisses, the minor term cannot therefore be distributed, and it follows that the conclusion must be particular.

If one of the premisses is negative, we may have three terms distributed in the premisses ; these must, however, be the middle term twice (by hypothesis), and the major term (since the conclusion must now be negative and the major term will therefore be distributed in it); hence the minor term cannot be distributed in the premisses, and it again follows that the conclusion must be particular.

But either both premisses will be affirmative, or one affirmative and the other negative ; in any case, therefore, we can infer that the conclusion will be particular.

[This proof seems preferable to that given by Jevons, *Studies in Deductive Logic*, p. 83.]

140. Shew that if the conclusion of a syllogism be a universal proposition, the middle term can be but once distributed in the premisses.   [L.]

As pointed out by Professor Jevons (*Studies in Deductive Logic*, p. 85), this proposition is the contrapositive of the result obtained in the preceding section.

141. Shew *directly* in how many ways it is possible to prove the conclusions SaP, SeP; point out those that conform immediately to the *Dictum de omni et nullo ;* and exhibit the equivalence between these and the remainder.   [W.]

(1)   To prove *All S is P.*

Both premisses must be affirmative, and both must be universal.

S being distributed in the conclusion, must be distributed in the minor premiss, which must therefore be *All S is M.*

M not being distributed in the minor must be distributed in the major which must therefore be *All M is P.*

SaP can therefore be proved in only one way, namely,

<div style="text-align:center">

All *M* is *P*,

All *S* is *M*,

therefore, All *S* is *P*;

</div>

and this syllogism conforms immediately to the *Dictum*.

(2) To prove *No S is P*.

Both premisses must be universal, and one must be negative while the other is affirmative, *i. e.*, one premiss must be *E* and the other *A*.

First, let the major be *E*, *i. e.*,

<div style="text-align:center">either *No M is P* or *No P is M*.</div>

In each case the minor must be affirmative and must distribute *S*; therefore, it will be *All S is M*.

Secondly, let the minor be *E*, *i. e.*,

<div style="text-align:center">either *No M is S* or *No S is M*.</div>

In each case the major must be affirmative and must distribute *P*; therefore, it will be *All P is M*.

We can then prove *SeP* in four ways, thus,—

(i) *MeP*,	(ii) *PeM*,	(iii) *PaM*,	(iv) *PaM*,
SaM,	*SaM*,	*MeS*,	*SeM*,
SeP.	*SeP*.	*SeP*.	*SeP*.

Of these, (i) only conforms immediately to the *Dictum*, and we have to shew the equivalence between it and the others.

The only difference between (i) and (ii) is that the major premiss of the one is the simple converse of the major premiss of the other; they are therefore equivalent. Similarly the only difference between (iii) and (iv) is that the minor premiss of the one is the simple converse of the

minor premiss of the other; they are therefore equivalent.

Finally, we may shew that (iii) is equivalent to (i) by transposing the premisses and converting the conclusion.

142. Shew *directly* in how many ways it is possible to prove the conclusions SiP, SoP.   [w.]

CHAPTER III.

143. Figure and Mood.

By the *Figure* of a Syllogism is meant the position of the terms in the premisses.

Denoting the major, middle and minor terms by the letters P, M, S respectively, and stating the major premiss first, we have four figures of the syllogism as shewn in the following table:—

Fig. 1.	Fig. 2.	Fig. 3.	Fig. 4.
$M - P$	$P - M$	$M - P$	$P - M$
$S - M$	$S - M$	$M - S$	$M - S$
$S - P$	$S - P$	$S - P$	$S - P.$

By the *Mood* of a Syllogism is meant the quantity and quality of the premisses and conclusion. Thus AAA, EIO are different moods. It is clear that if figure and mood are both given, the syllogism is given.

144. The Special Rules of the Figures; and the Determination of the Legitimate Moods in each Figure[1].

---

[1] The method of Determination here adopted is only one amongst several possible methods. Another is suggested, for example, in sections 141, 142.

It may first of all be shewn that certain combinations of premisses are incapable of yielding a valid conclusion in any figure. *A priori*, there are possible the following sixteen different combinations of premisses, the major premiss being always stated first:—*AA, AI, AE, AO, IA, II, IE, IO, EA, EI, EE, EO, OA, OI, OE, OO.* Referring back however to the syllogistic rules (section 114), we find that of these, *EE, EO, OE, OO,* (being combinations of negative premisses), give no conclusion by rule 5; again, *II, IO, OI,* (being combinations of particular premisses), are excluded by corollary i.; and *IE* is excluded by corollary iii., which tells us that nothing follows from a particular major and a negative minor.

We are left then with the following eight possible combinations:—*AA, AI, AE, AO, IA, EA, EI, OA*; and we may now go on to determine in which figures these will yield conclusions.

The special rules and the legitimate moods of Figure 1.

The position of the terms in Figure 1 is shewn thus,—

$$M - P$$
$$S - M$$
$$\overline{S - P}$$

and we can prove that in this figure :—

(1) *The minor premiss must be affirmative.* For if it were negative, the major premiss would have to be affirmative by rule 5, and the conclusion negative by rule 6. The major term would therefore be distributed in the conclusion, and undistributed in its premiss; and the syllogism would be invalid by rule 4.

(2) *The major premiss must be universal.* For the middle term cannot be distributed in the minor premiss since this

is affirmative, and must therefore be distributed in the major premiss.

Rule (1) shews that AE and AO, and rule (2) that IA and OA yield no conclusions in this figure. We are therefore left with only four combinations, namely, AA, AI, EA, EI. Applying the rules that a negative premiss gives a negative conclusion, while conversely a negative conclusion requires a negative premiss, and that a particular premiss gives a particular conclusion only, we find that AA will justify either of the conclusions A or I, EA either E or O, AI only I, EI only O. We have then six moods in Figure 1 which do not offend against any of the rules of the syllogism, namely, AAA, AAI, AII, EAE, EAO, EIO.

We may establish the actual validity of these moods by shewing that the axiom of the syllogism, the *Dictum de omni et nullo*, applies to them; or by taking them severally and shewing that in each case the cogency of the reasoning is self-evident.

The special rules and the legitimate moods of Figure 2.

The position of the terms in figure 2 is shewn thus,—

$$\begin{array}{c} P - M \\ \underline{S - M} \\ S - P \end{array}$$

and its special rules, (which the student is recommended to deduce from the general rules of syllogism for himself), are,—

(1) *One premiss must be negative;*

(2) *The major premiss must be universal.*

Applying these rules, we shall find that we are again left with six moods, namely, AEE, AEO, AOO, EAE, EAO, EIO.

We cannot now apply the *Dictum de omni et nullo* to shew positively that these moods are legitimate. We may however as before establish the cogency of the reasoning in each case by shewing it to be self-evident. The older logicians did not adopt this course, but they proved that by means of immediate inferences each could be reduced to such a form that the *Dictum* could be directly applied to it. This is the doctrine of Reduction to which reference will be made subsequently.

The special rules and the legitimate moods of Figure 3.

The position of the terms in this figure is shewn thus,—

$$\frac{\begin{array}{c} M - P \\ M - S \end{array}}{S - P}$$

and its special rules are,—

(1) *The minor must be affirmative;*

(2) *The conclusion must be particular.*

Proceeding as before, we shall find ourselves left with six valid moods,—*AAI, AII, EAO, EIO, IAI, OAO.*

The special rules and the legitimate moods of Figure 4.

The position of the terms in this figure is shewn thus,—

$$\frac{\begin{array}{c} P - M \\ M - S \end{array}}{S - P}$$

and its special rules are,—

(1) *If the major is affirmative, the minor must be universal;*

(2) *If either premiss is negative, the major must be universal;*

(3) *If the minor is affirmative, the conclusion must be particular.*

The result of the application of these rules is again six valid moods:—*AAI, AEE, AEO, EAO, EIO, IAI.*

Our final conclusion then is that there are 24 valid moods, namely, six in each figure.

In Figure 1, *AAA, AAI, EAE, EAO, AII, EIO.*

In Figure 2, *EAE, EAO, AEE, AEO, EIO, AOO.*

In Figure 3, *AAI, IAI, AII, EAO, OAO, EIO.*

In Figure 4, *AAI, AEE, AEO, EAO, IAI, EIO.*

145. Weakened Conclusions, and Subaltern Moods.

When from premisses that would have justified a universal conclusion we content ourselves with inferring a particular, (as, for example, in the syllogism All *M* is *P*, All *S* is *M*, therefore, Some *S* is *P*), we are said to have a *weakened conclusion*, and the syllogism is said to be a *weakened syllogism* or to be in a *subaltern mood,* (because the conclusion might be obtained by subaltern opposition from the conclusion of the corresponding strong mood).

In the preceding section it has been shewn that in each figure there are six moods which do not offend against any of the syllogistic rules ; so that in all we should have 24 distinct valid moods. Five of these however have weakened conclusions ; and, since we are not likely to be satisfied with a particular conclusion when the corresponding universal could be obtained from the same premisses, these moods are of no practical importance, so that when the moods of the various figures are enumerated (as in the mnemonic verses) they are usually omitted.

The subaltern moods are,—

In Figure 1, *AAI, EAO ;*

In Figure 2, *EAO, AEO ;*

In Figure 4, *AEO.*

146. In what figure can there be no weakened conclusion and why ? Do any of the 19 moods commonly recognised give a weaker conclusion than the premisses would warrant ?   [L.]

It is obvious that there can be no weakened conclusion in Figure 3, since in no case can we infer more than a particular conclusion in this figure.

I should answer the question, "whether any of the 19 moods commonly recognised yield a weaker conclusion than the premisses would warrant," in the negative. Professor Jevons (*Studies in Deductive Logic*, p. 87) apparently answers it in the affirmative, having in view *AAI* in Figure 4.

With the premisses

All *P* is *M*,
All *M* is *S* ;

the conclusion Some *S* is *P* is certainly in one sense weaker than the premisses would warrant since we might have inferred the universal conclusion All *P* is *S*. But All *P* is *S* is not the universal corresponding to Some *S* is *P*. The subjects of these two propositions are different ; and we infer all that we possibly can about *S* when we say Some *S* is *P*. In other words, regarded as a mood of Figure 4, this mood is not a subaltern. *AAI* in Figure 4 is thus differentiated from *AAI* in Figure 1, and its recognition in the mnemonic verses justified.

I do not quite understand Professor Jevons's comments on this case. Answering the same question as that with

which we are dealing, he says "*Bramantip*[1] of the fourth
figure is the single mood alluded to in the latter part of the
question. Considering that it is impossible to employ con-
version by limitation without weakening the logical force of
the premiss, it is too bad of the Aristotelian logicians to
slight the weakened moods of the syllogism as they have
usually done" (*Studies*, pp. 87, 88). The truth is that for
practical purposes they may certainly be neglected[2]; but
their recognition gives a completeness to the theory of Syl-
logism which it cannot otherwise possess. There is also a
symmetry in the result of their recognition as yielding
exactly six legitimate moods in each figure.

147. Strengthened Syllogisms.

If in a syllogism, the same conclusion could be obtained
although we substituted for one of the premisses its sub-
altern, the syllogism is said to be a *strengthened syllogism*.
A strengthened syllogism is thus a syllogism with an un-
necessarily strengthened premiss.

For example, the conclusion of the syllogism,—

<div style="text-align:center">

All M is P,

All M is S,

</div>

therefore,   Some S is P,

could equally be obtained from the premisses,—

<div style="text-align:center">

All M is P,

Some M is S;

</div>

or from the premisses,—

<div style="text-align:center">

Some M is P,

All M is S.

</div>

----

[1] *i.e.*, *AAI* in Figure 4. Cf. section 158.

[2] *AAI* in Figure 4 is not to be regarded as a weakened mood, as I
have just shewn.

By trial we may find that *every syllogism in which there are two universal premises with a particular conclusion is a strengthened syllogism, with the one exception of AEO in the fourth Figure*[1].

In a full enumeration there are two strengthened syllogisms in each figure :—

> In Figure 1, *AAI, EAO* ;
> In Figure 2, *EAO, AEO* ;
> In Figure 3, *AAI, EAO* ;
> In Figure 4, *AAI, EAO*.

The distinction between a strengthened syllogism, (that is, a syllogism with a strengthened premiss), and a weakened syllogism, (that is, a syllogism with a weakened conclusion), should be carefully noted.

It will be observed that in Figures 1 and 2, a syllogism having a strengthened premiss may also be regarded as a syllogism having a weakened conclusion, and *vice versa;* but in Figures 3 and 4, the contradictory holds in both cases. The only syllogism with a weakened conclusion in either of these figures is *AEO* in Figure 4, but this does not contain a strengthened premiss. That is, having

> All *P* is *M*,
> No *M* is *S*,
> ──────────
> therefore, Some *S* is not *P* ;

the syllogism becomes invalid, if for either of the premisses we substitute its subaltern.

148. The peculiarities and uses of each of the four figures of the syllogism.

Figure 1. In this figure we can prove conclusions of all the forms *A, E, I, O* ; and it is the *only* figure in which

───────────

[1] A general proof of this proposition is given in section 281.

we can prove a universal affirmative conclusion. This alone makes it by far the most useful and important of the syllogistic figures. All deductive science, the object of which is to establish universal affirmatives, tends to work in *AAA* in this figure.

Another point to notice is that only in this figure have we both the subject of the conclusion as subject in the premisses, and the predicate of the conclusion as predicate in the premisses. (In Figure 2 the predicate of the conclusion is subject in the major premiss; in Figure 3 the subject of the conclusion is predicate in the minor premiss; and in Figure 4 we have a double inversion.) This is no doubt one reason why reasoning in Figure 1 so often seems more natural than the same reasoning expressed in any of the other figures[1].

Figure 2. In this figure we can prove negatives only; and therefore it is chiefly used for purposes of disproof. For example, Every real natural poem is naïve; those poems of Ossian which Macpherson pretended to discover are not naïve (but sentimental); hence they are not real natural poems. (Ueberweg, *System of Logic*, translation by Lindsay, p. 416.) It has been called the *exclusive* figure; because by means of it we may go on excluding various suppositions as to the nature of something under investigation, whose real character we wish to ascertain, (a process called *abscissio infiniti*).

For example,

> Such and such an order has such and such properties,
> This plant has not those properties;

therefore, It does not belong to that order.

---

[1] Compare Solly, *Syllabus of Logic*, pp. 130—132.

This syllogism might be repeated with a number of different orders till the enquiry is so narrowed down that the place of the plant is easily determined.   Whately (*Elements of Logic*, p. 92) gives an example from the diagnosis of a disease.

Figure 3.   In this figure we can prove particulars only. It is frequently useful when we wish to take objection to a universal proposition laid down by an opponent, by establishing an instance in which such universal proposition does not hold good.

It is the natural figure when the middle term is a singular term, especially if the other terms are general.   We have already shewn that if one and only one term of an affirmative proposition is singular it is almost necessarily the subject. For example, such a reasoning as,—

> Socrates was wise,
> Socrates was a philosopher,

therefore,   Some philosophers are wise,

could only be expressed with great awkwardness in any figure other than Figure 3.

Figure 4.   This figure is seldom used, and some logicians have altogether refused to recognise it.   We shall return to a discussion of it subsequently.   Compare section 172.

[Lambert, (a distinguished mathematician as well as logician, whose *Neues Organon* appeared in 1764), expressed the uses of the different syllogistic figures as follows : "The first figure is suited to the discovery or proof of the properties of a thing ; the second to the discovery or proof of the distinctions between things ; the third to the discovery or proof of instances and exceptions ; the fourth to the discovery or exclusion of the different species of a genus."

De Morgan (*Syllabus*, p. 30) thus characterizes the different figures,—

"The first figure may be called the figure of *direct transition*: the fourth, which is nothing but the first with a *converted conclusion*[1], the figure of *inverted transition;* the second, the figure of *reference to* (the middle term); the third, the figure of *reference from* (the middle term).")

149. Shew the inadequacy of Hamilton's proof of the special rule that in Figure 2 one premiss must be negative. "For were there two affirmative premisses, as :—

> All *P* are *M* ;
> All *S* are *M* ;
> *All metals are minerals ;*
> *All pebbles are minerals ;*

the conclusion would be—*All pebbles are metals,* which would be false" (*Logic*, vol. I, pp. 408, 9).

150. Which of the following conjunctions of propositions make valid syllogisms? In the case of those which you regard as invalid, give your reasons for so treating them.

Fig. 1.	Fig. 2.	Fig. 3.	Fig. 4.
AEE	*AAA*	*AOE*	*AII*
AOO	*AOE*	*AEO*	
	IEA	*AOO*	
	AEE	*IEO*	
	AAI		

[c.]

---

[1] Cf. section 172.

151. What Moods are good in the first figure and faulty in the second, and *vice versa*? Why are they excluded in one figure and not in the other?

[O.]

152. Shew that O cannot stand as premiss in Figure 1, as major in Figure 2, as minor in Figure 3, as premiss in Figure 4.   [C.]

153. Shew that it is impossible to have the conclusion in A in any figure but the first. What fallacies would be committed if there were such a conclusion to a reasoning in any other figure?   [C.]

154. Shew that a syllogism in Figure 4 cannot have O for a premiss, nor A for a conclusion.   [C.]

155. Prove that in Figure 4, if the minor premiss is negative, both the premisses must be universal.

CHAPTER IV.

THE REDUCTION OF SYLLOGISMS.

156. The Problem of Reduction.

By *Reduction* is meant the process of expressing the reasoning contained in a given syllogism in some other mood or figure. Unless otherwise stated, Reduction is always supposed to be to Figure 1.

As an example, we may take the following syllogism in Figure 3,—

<div align="center">

All M is P,

Some M is S,

therefore, Some S is P.

</div>

It will be seen that by simply converting the minor premiss, we have precisely the same reasoning in Figure 1.

This is an example of *direct* or *ostensive* reduction.

157. Indirect Reduction.

We prove a proposition *indirectly* when we prove its contradictory to be false; and this may be done by shewing that an ultimate consequence of the truth of its contradictory is the truth of some proposition that is self-evidently false.

The method of indirect proof is in several cases adopted by Euclid; and it is sometimes employed in the reduction

of syllogisms from one mood to another. Thus, AOO in Figure 2 is usually reduced in this manner. From the premisses,—

<div align="center">

All P is M,

Some S is not M,

</div>

it follows that   Some S is not P;

for if this conclusion is not true, its contradictory (namely, All S is P), must be so, and the premisses being given true we shall have true together the three propositions,—

<div align="center">

All P is M; (1)

Some S is not M; (2)

All S is P.   (3)

</div>

But combining (1) and (3) we have a syllogism in Figure 1,—

<div align="center">

All P is M,

All S is P,

</div>

yielding the conclusion All S is M.   (4)

Some S is not M (2), and All S is M (4) are therefore true together; but this is self-evidently absurd, since they are contradictories.

Hence it has been shewn that the consequence of supposing Some S is not P false is a self-contradiction; and we may therefore infer that it is true.

It will be observed that the only explicit syllogism that has been made use of in the above is in Figure 1 [1]; and the

---

[1] Solly (*Syllabus of Logic*, p. 104) maintains that a full analysis of the reasoning will shew that three distinct syllogisms are really involved,—

" Let A and B represent the premisses, and C the conclusion of any syllogism. In order to prove C by the indirect method, we commence with assuming that C is not true. The three syllogisms may be then stated as follows :

process is therefore regarded as a reduction of the reasoning to Figure 1.

This method of reduction is called *Reductio ad impossibile*, or *Reductio per impossibile*[1], or *Deductio ad impossibile*, or *Deductio ad absurdum*. It is the only way of reducing *AOO* (Figure 2), or *OAO* (Figure 3), to Figure 1, unless we make use of negative terms (as in obversion and contraposition); and it was adopted by the old writers in consequence of their objection to negative terms.

158. The mnemonic lines *Barbara, Celarent*, &c.

The mnemonic verses, (which are spoken of by De Morgan as "the magic words by which the different moods have been denoted for many centuries, words which I take to be more full of meaning than any that ever were made"), are usually given as follows,—

*Barbara, Celarent, Darii, Ferio*que prioris:
Cesare, Camestres, Festino, Baroco, secundae:
Tertia, *Darapti, Disamis, Datisi, Felapton,*
Bocardo, Ferison, habet: Quarta insuper addit
Bramantip, Camenes, Dimaris, Fesapo, Fresison.

Each valid mood in every figure, unless it be a subaltern

First syllogism: '*A* is; *C* is not; therefore *B* is not'.

Second syllogism: 'If *A* is, and *C* is not, it follows that *B* is not; but *B* is; therefore it is false that *A* is and *C* is not."

Third syllogism: 'Either both propositions *A* is and *C* is not are false, or else one of them is false; but that *A* is is not false; therefore that *C* is not is false, (*i. e.*, *C* is).'"

I do not see any flaw in this analysis; at any rate it must be admitted that the reasoning involved in Indirect Reduction is highly complex, and since the two moods to which it is generally applied can also be reduced directly (compare section 159), some modern logicians are inclined to banish it entirely from their treatment of the syllogism.

[1] Cf. Mansel's *Aldrich*, pp. 88, 89.

mood, is here represented by a separate word ; and in the case of a mood in any of the so-called imperfect figures, (*i.e.*, Figures 2, 3, 4), the mnemonic gives full information for its reduction to Figure 1, the so-called perfect figure.

The only meaningless letters are *b* (not initial), *d, l, n, r, t* ; the signification of the remainder is as follows :—

The vowels give the quality and quantity of the propositions of which the syllogism is composed ; and therefore really give the syllogism itself. Thus, *Camenes* being in Figure 4, represents the syllogism,—

$$\text{All } P \text{ is } M,$$
$$\text{No } M \text{ is } S,$$
$$\text{therefore,} \quad \text{No } S \text{ is } P.$$

The initial letters in the case of Figures 2, 3, 4 shew to which of the moods of Figure 1 the given mood is to be reduced, namely to that which has the same initial letter. [The letters *B, C, D, F* were chosen for the moods of Figure 1 as being the first four consonants in the alphabet.]

Thus, *Camestres* is reduced to *Celarent*,—

All *P* is *M*,	$\times$	No *M* is *S*,
No *S* is *M*,		All *P* is *M*,

therefore,   No *S* is *P*. therefore, No *P* is *S*,
                              therefore, No *S* is *P*.

s (in the middle of a word) indicates that in the process of reduction the preceding proposition is to be simply converted. Thus, in reducing *Camestres* to *Celarent*, as shewn above, the minor premiss is simply converted.

s (at the end of a word[1]) shews that the conclusion of the *new* syllogism has to be simply converted in order to

---

[1] This slight difference in the signification of *s* and *p* when they are *final* letters is frequently overlooked.

obtain the given conclusion. This again is illustrated in the reduction of *Camestres*. The final *s* does not affect the conclusion of *Camestres* itself, but the conclusion of *Celarent* to which it is reduced.

p (in the middle of a word) signifies that the preceding proposition is to be converted *per accidens*. Thus, in the reduction of *Darapti* to *Darii*,—

All M is P,	All M is P,
All M is S,	Some S is M,

therefore, Some S is P. therefore, Some S is P.

p (at the end of a word[1]) implies that the conclusion *obtained by reduction* is to be converted *per accidens*. Thus, in *Bramantip*, the *p* obviously cannot affect the I conclusion of the mood itself; it really affects the A conclusion of the syllogism in *Barbara* which is given by reduction. Thus,—

All P is M,		All M is S,
All M is S,	$\times$	All P is M,

therefore, Some S is P. therefore, All P is S,
therefore, Some S is P.

m indicates that in reduction the premisses have to be transposed, (*Metathesis præmissarum*) ; as just shewn in the case of *Bramantip*.

c signifies that the mood is to be reduced *indirectly*, (*i.e.*, by *reductio per impossibile* in the manner indicated in the preceding section) ; and the position of the letter indicates that in this process of indirect reduction the first step is to omit the premiss preceding it, *i.e.*, the other premiss is to be combined with the contradictory of the conclusion, (*Conversio syllogismi*, or *ductio per Contradictoriam propositionem sive per impossibile*). *c* is by some writers replaced by *k*, thus *Baroko* and *Bokardo* instead of *Baroco* and *Bocardo*.

[1] See note on the preceding page.

The following lines are sometimes added to the verses given above, in order to meet the case of the subaltern moods:—

> Quinque Subalterni, totidem Generalibus orti,
> Nomen habent nullum, nec, si bene colligis, usum[1].

159. The direct reduction of *Baroco* and *Bocardo*. Mnemonics representing the direct reduction of these moods.

---

[1] The mnemonics have been written in various forms. Those given above are from Aldrich, and they are the ones that are in general use in England. Wallis in his *Institutio Logicæ* (1687) gives for Figure 4, *Balani, Cadere, Digami, Fegano, Fedibo*. P. van Musschenbroek in his *Institutiones Logicæ* (1748) gives *Barbari, Calentes, Dibatis, Fespamo, Fresisom*. This variety of forms for the moods of Figure 4 was no doubt due to the fact that the recognition of this figure at all was quite exceptional until comparatively recently. Compare section 173.

According to Ueberweg, the mnemonics run,—

> *Barbara, Celarent* primæ, *Darii Ferio*que.
> *Cesare, Camestres, Festino, Baroco* secundæ.
> Tertia grande sonans recitat *Darapti, Felapton,*
> *Disamis, Datisi, Bocardo, Ferison.* Quartæ
> Sunt *Bamalip, Calemes, Dimatis, Fesapo, Fresison.*

Mr Carveth Read (*Mind*, 1882, p. 440) suggests an ingenious modification of the verses, so as to make each mnemonic immediately suggest the figure to which the mood it represents belongs, at the same time abolishing all the unmeaning letters. He takes *l* as the sign of the first figure, *n* of the second, *r* of the third, and *t* of the fourth. The lines then run

> *Ballala, Celallel, Dalii, Felio*que prioris.
> *Cesane, Camesnes, Fesinon, Banoco* secundæ.
> Tertia *Darapri, Drisamis, Darisi, Ferapro,*
> *Bocaro, Ferisor* habet. Quarta insuper addit
> *Bamatip, Cametes, Dimatis, Fesapto, Fesistot.*

Mr Read also suggests mnemonics to indicate the direct reduction of Baroco and Bocardo. Compare the following section.

Baroco :—

All *P* is *M*,
Some *S* is not *M*,

therefore, Some *S* is not *P*,

may be reduced to *Ferio* by contrapositing the major pre
miss, and obverting the minor premiss, thus,—

No not-*M* is *P*,
Some *S* is not-*M*,

therefore, Some *S* is not *P*.

Professor Croom Robertson has suggested *Faksoko* to
represent this method of reduction, *k* denoting obversion, so
that *ks* denotes obversion followed by conversion, (*i.e.*, con-
traposition).

Whately's word *Fakoro* (*Elements of Logic*, p. 97) does
not indicate the obversion of the minor premiss (*r* being
with him an unmeaning letter).

Bocardo :—

Some *M* is not *P*,
All *M* is *S*,

therefore, Some *S* is not *P*,

may be reduced to *Darii* by contrapositing the major
premiss and transposing the premisses, thus,

All *M* is *S*,
Some not-*P* is *M*,

therefore, Some not-*P* is *S*.

We have first to convert and then to obvert this conclu-
sion, however, in order to get the original conclusion. This
process may be indicated by *Doksamosk*, (which again is
obviously preferable to *Dokamo* suggested by Whately,

since this word would make it appear as if we immediately
obtained the original conclusion in *Darii*[1]).

160. Shew how to reduce *Bramantip* by the
indirect method.

Just as *Bocardo* and *Baroco* which are usually reduced
indirectly may be reduced directly, so other moods which
are usually reduced directly may be reduced indirectly.

Bramantip :—

> All *P* is *M*,
> All *M* is *S*,
> therefore, Some *S* is *P* ;

for, if not, then No *S* is *P*; and combining this with the
given minor premiss we have a syllogism in *Celarent*,—

> No *S* is *P*,
> All *M* is *S*,
> therefore, No *M* is *P*,

which yields by conversion No *P* is *M*. But this is the
contrary of the original major premiss All *P* is *M*, and it is
impossible that they should be true together. Hence we
infer the truth of the original conclusion.

161. Assuming that any syllogistic reasoning can
be expressed in the first Figure, *prove* that, (omitting
the subaltern moods), it can be expressed, directly or
indirectly, in any given mood of that Figure.

---

[1] Mr Carveth Read (*Mind*, 1882, p. 441) uses the letters *k* and *s* as
above; but his mnemonics are required also to indicate the figure to
which the moods belong (compare the preceding note); and he there-
fore arrives at *Faksnoko* and *Doksamrosk*.

Spalding (*Logic*, p. 235) suggests *Facoco* and *Docamoc*; but the
processes here indicated by the letter *c* are not in all cases the same,
and these mnemonics are therefore unsatisfactory.

We may extend the doctrine of reduction, and shew not merely that any syllogism may be reduced to Figure 1, but also that it may be reduced to any given mood of that figure, provided it is not a subaltern mood. This position will obviously be established if we can shew that *Barbara*, *Celarent*, *Darii* and *Ferio* are mutually reducible to one another. *Barbara* may be reduced to *Celarent* by obverting the major premiss and also the new conclusion which is thereby obtained. Thus,

$$\text{All } M \text{ is } P,$$
$$\text{All } S \text{ is } M,$$

therefore, All S is P,
becomes No M is not-P,
$$\text{All } S \text{ is } M,$$

therefore, No S is not-P,
therefore, All S is P.

Conversely, *Celarent* is reducible to *Barbara*; and in a similar manner by obversion of major premiss and conclusion *Darii* and *Ferio* are reducible to each other.

It will now suffice if we can shew that *Barbara* and *Darii* are mutually reducible to each other. Obviously the only method possible here is the *indirect* method.

Take *Barbara*, MaP,
                               SaM,
                         ─────
                          SaP;

for, if not, then we have SoP; and MaP, SaM, SoP must be true together. From SoP by first obverting and then converting, (and denoting not-P by P'), we get $P'iS$, and combining this with SaM we have a syllogism in *Darii*,—

$$Sa M,$$
$$\frac{P'iS,}{P'iM.}$$

$P'iM$ by conversion and obversion becomes MoP; and therefore MaP and MoP are true together; but this is impossible, since they are contradictories.

Therefore, SoP cannot be true, *i.e.*, the truth of SaP is established.

Similarly, *Darii* may be indirectly reduced to *Barbara*[1].

$$MaP, \quad \text{(i)}$$
$$\frac{SiM, \quad \text{(ii)}}{SiP. \quad \text{(iii)}}$$

The contradictory of (iii) is SeP, from which we obtain PaS'. Combining with (i), we have—

$$PaS'$$
$$\frac{MaP,}{MaS' \text{ in } Barbara.}$$

But from this conclusion we may obtain SeM, which is the contradictory of (ii)[2].

162. Some logicians have asserted that all the moods of the syllogism are reducible to the form of Barbara. Inquire into the truth of this assertion. [L.]

163. Making use of any legitimate methods of immediate inference that may be serviceable, shew

[1] It has also been maintained, that this reduction is unnecessary, and that, to all intents and purposes, *Darii is Barbara*, since the "some S" in the minor is, and is known to be, the *same some* as in the conclusion.

[2] It would now seem that the *Dictum de omni et nullo* might if we pleased be reduced to a *Dictum de omni;* but it would be vain to pretend that any real simplification would be introduced thereby.

how *Barbara*, *Baroco* and *Bocardo* may be reduced ostensively to Figure 4.

164. Reduce *Ferio* to Figure 2, *Festino* to Figure 3, *Felapton* to Figure 4.

165. Prove that any mood may be reduced to any other mood provided that the latter contains neither a strengthened premiss nor a weakened conclusion.

166. Examine the following statement of De Morgan's :—" There are but six distinct syllogisms. All others are made from them by strengthening one of the premisses, or converting one or both of the premisses, where such conversion is allowable ; or else by first making the conversion, and then strengthening one of the premisses."

167. How can you apply the *Dictum de omni et nullo* to the following syllogism :—Some *M* is not *P*, All *M* is *S*, therefore, Some *S* is not *P* ?

168. How would you apply the *Dictum de omni et nullo* to the following reasonings ?

(1) The life of St Paul proves the falsity of the conclusion that only the rich are happy.

(2) His weakness might have been foretold from his proneness to favourites, for all weak princes have that failing.   [V.]

169. *Dicta* for the second and third Figures of syllogism corresponding to the *Dictum* of the first.

Thomson (*Laws of Thought*, p. 173), and Bowen (*Logic*, p. 196), give for Figure 2, a *dictum de diverso*,—"If one

term is contained in, and another excluded from, a third term, they are mutually excluded"; and for Figure 3, a *Dictum de exemplo*,—"Two terms which contain a common part, partly agree, or if one one contains a part which the other does not, they partly differ." The former of these is at least expressed loosely since it would appear to warrant a universal conclusion in *Festino* and *Baroco*. Mansel (*Aldrich*, p. 86) puts this *Dictum* in a more satisfactory form:—"If a certain attribute can be predicated, affirmatively or negatively, of every member of a class, any subject of which it cannot be so predicated, does not belong to the class." This proposition may claim to be axiomatic, and it can be applied directly to any syllogism in Figure 2.

The *Dictum de exemplo* again as stated above is open to exception. The proposition, "If one term contains a part which another does not they partly differ," applied to No M is P, All M is S, would appear to justify Some P is not S just as much as Some S is not P. Mansel's amendment here is to give two principles for Figure 3, the *Dictum de exemplo*,—"If a certain attribute can be affirmed of any portion of the members of a class, it is not incompatible with the distinctive attributes of that class"; and the *Dictum de excepto*,—"If a certain attribute can be denied of any portion of the members of a class, it is not inseparable from the distinctive attributes of that class." But is it essential that in the minor premiss we should be predicating the *distinctive attributes* of the class as is here implied? This appears to be a fatal objection to Mansel's *dicta* for Figure 3. Moreover, granted that P is *not incompatible* with S, are we therefore justified in saying Some S is P?

I would suggest the following axioms,—"If two terms are both affirmatively predicated of a common third, and one at least of them universally so, they may be par-

tially predicated of each other"; "If one term is denied while another is affirmed of a common third term, either the denial or the affirmation being universal, the former may be partially denied of the latter." These will I think be found to apply respectively to the affirmative and negative moods of Figure 3, and they may be regarded as axiomatic; but they are certainly somewhat laboured.

170. Is *Reduction* an essential part of the doctrine of the syllogism?

According to the original theory of Reduction, the object of the process was to be sure that the conclusion was a valid inference from the premises. Given a syllogism in Figure 1, we are able to test its validity by reference to the *Dictum de omni et nullo;* but we have no such means of dealing directly with syllogisms in any other figure. Thus, Whately says,— "As it is on the *Dictum de omni et nullo* that all Reasoning *ultimately* depends, so, all arguments may be in one way or other brought into some one of the four Moods in the First Figure: and a Syllogism is, in that case, said to be *reduced*" (*Elements of Logic*, p. 93). Professor Fowler puts the same position in a more guarded manner,—"As we have adopted no canon for the 2nd, 3rd, and 4th figures, we have as yet no positive proof that the six moods remaining in each of those figures are valid; we merely know that they do not offend against any of the syllogistic rules. But if we can *reduce* them, *i.e.*, bring them back to the 1st figure, by shew- ing that they are only different statements of its moods, or in other words, that precisely the same conclusions can be obtained from equivalent premises in the 1st figure, their validity will be proved beyond question" (*Deductive Logic*, p. 97).

On the other hand, by some logicians Reduction is

regarded as *unnecessary* and *unnatural*. It is maintained to
be *unnecessary* on the ground that it is not true that the
Dictum de omni et nullo is the paramount law for all perfect
inference, or that the first figure is alone perfect[1]. In the
preceding section we have discussed *dicta* for the other
figures, which may be regarded as making them independent
of the first, and putting them on a level with it. It may
also be maintained that in any mood the validity of a par-
ticular syllogism is as self-evident as that of the *Dictum*
itself; and that therefore although axioms of syllogism are
useful as *generalisations* of the syllogistic process, they are
needless in order to establish the validity of any given syllo-
gism. . This view is indicated by Ueberweg.

Again, Reduction is said to be *unnatural*, inasmuch as
it often involves the substitution of an unnatural and indirect
for a natural and direct predication. Figures 2 and 3 at
any rate have their special uses, and certain reasonings
naturally fall into these figures rather than into Figure 1.
This argument is very well elaborated by Archbishop
Thomson (*Laws of Thought*, pp. 173—175). He gives this
example,—"Thus, when it was desirable to shew by an
example that zeal and activity did not always proceed from
selfish motives, the natural course would be some such syl-
logism as the following. The Apostles sought no earthly
reward, the Apostles were zealous in their work; therefore,
some zealous persons seek not earthly reward." In reducing
this syllogism to Figure 1, we have to convert our minor into
"Some zealous persons were Apostles," which is awkward
and unnatural.

Take again this syllogism,

"Every reasonable man wishes the Reform Bill to pass,

---

[1] Cf. Thomson, *Laws of Thought*, p. 172.

I don't,
therefore, I am not a reasonable man."

Reduced in the regular way to *Celarent*, the major pre-
miss becomes "No person wishing the Reform Bill to pass
is I," yielding the conclusion, "No reasonable man is I."

Further illustrations of this point will be found if we
reduce to Figure 1, syllogisms with such premisses as the
following :—

> { Bashfulness is not praiseworthy,
>   Modesty is praiseworthy.

> { Socrates is poor,
>   Socrates is wise.

The above arguments appear conclusively to establish
the view that Reduction is not an essential part of the doc-
trine of Syllogism, at any rate so far as establishing the
validity of the different moods is concerned.

It may, however, be doubted whether any treatment of
the Syllogism can be regarded as scientific or complete until
the *equivalence* between the moods in the different figures
has been shewn; and for this purpose, as well as for its
utility as a logical exercise, a full treatment of the problem
of Reduction should be retained.

171. Discuss Hamilton's doctrine that Figures 2,
3, and 4, are not genuine and original forms of reason-
ing.

"The last three figures," says Hamilton (*Logic*, I. p. 433),
"are virtually identical with the first." This has been recog-
nised by logicians, and hence "the tedious and disgusting
rules of their reduction." He himself however goes further,
and extinguishes these figures altogether, as being merely

"accidental modifications of the first," and "the mutilated expressions of a complex mental process."

If the last three figures are admitted as genuine and original forms of reasoning, the following anomalies in Hamilton's opinion result:—

"In the first place, the principle that all reasoning is the recognition of the relation of a least part to a greatest whole, through a lesser whole or greater part, is invalidated." In reply to this, it may simply be asked whether it really requires the last three figures to invalidate this principle.

"In the second place, the second general rule I gave you for categorial syllogisms, is invalidated in both its clauses." It does not occur to Hamilton that his rules may have been needlessly limited in their application. The fact is that he has with a great flourish of trumpets *simplified* the rules of the syllogism by replacing those usually given by *the special rules of Figure* 1 [1]; and he is now shocked to find that these do not apply to Figures 2, 3, 4. This whole reasoning of Hamilton's is a flagrant example of *petitio principii*.

The question at issue is really this,—can we formulate a principle which shall be accepted as axiomatic, and which shall apply immediately to syllogisms in other figures than the first?

---

[1] "Had Dr Whately looked a little closer into the matter, he might have seen that the six rules which he and other logicians enumerate, may, without any sacrifice of precision, and with even an increase of perspicuity, be reduced to three......These three simple laws comprise all the rules which logicians lay down with so confusing a minuteness" (Hamilton, *Logic*, I. pp. 305, 6). But as I have remarked in the text, the simplification is obtained solely by giving laws which have a more limited application than other logicians had contemplated.

Now take a syllogism in *Cesare*,—

$$\text{No } P \text{ is } M,$$
$$\text{All } S \text{ is } M,$$

therefore,   No S is P.

Hamilton maintains (*Logic*, I. pp. 434, 435) that we can only properly see the force of this reasoning by mentally converting the major premiss to No M is P. But will not the following which applies immediately to *Cesare* be accepted as axiomatic,—"If one class is excluded from and another is contained in a third class, the second class is excluded from the first"? This simple case seems to me sufficient to overthrow the whole of Hamilton's elaborate but confused reasoning[1].

Perhaps *Baroco* is a still better case,—

$$\text{All } P \text{ is } M,$$
$$\text{Some } S \text{ is not } M,$$

therefore,   Some S is not P.

Axiom: "If one class is totally contained in, and another partially excluded from a third class, the second class is partially excluded from the first." Now compare Hamilton's elaborate explication (*Logic*, I. pp. 438, 9),—" The formula of Baroco is :—

$$\text{All } P \text{ are } M;$$
$$\text{But some } S \text{ are not } M;$$
$$\text{Therefore, some } S \text{ are not } P.$$

But the following is the full mental process :—

Sumption,...................... All P are M;
Real Subsumption, ... (Some not-M are S);

---

[1] It may be pointed out that Hamilton himself elsewhere (*Logic*, II. p. 358) gives special Canons for Figures 2, 3.

which gives the

Expressed Subsumption, ... $\begin{cases}\text{Then, Some } S \text{ are not-}M; \\ \text{Or, Some } S \text{ are not } M;\end{cases}$

Real Conclusion,..............(Therefore, Some not-P are S);

which gives the

Expressed Conclusion, ... $\begin{cases}\text{Then, Some } S \text{ are not-}P: \\ \text{Or, Some } S \text{ are not } P.\end{cases}$"

It is surely absurd to say that we go through this complex mental process in order to discover the validity of a syllogism in *Baroco*.

But even granting that this is the case, I cannot see how on his own grounds Hamilton succeeds in getting rid of the necessity of "the tedious and disgusting" rules of reduction; nor that he has advanced beyond the logicians who reject the independent validity of Figures 2, 3, 4, and consequently establish the necessity of the process of reduction, and naturally along with it of rules for conducting the process. It would seem that only those logicians who, like Thomson, maintain the independent validity of other figures than the first have any justification whatever for ignoring the doctrine of reduction.

172. *The Fourth Figure.*

Figure 4 was not as such recognised by Aristotle ; and its introduction having been attributed by Averroes to Galen, it is frequently spoken of as the *Galenian* figure. It does not usually appear in works on logic before the beginning of the last century, and even by modern logicians its use is sometimes condemned. Thus, Bowen (*Logic*, p. 192) holds that "what is called the Fourth Figure is only the First with a converted conclusion ; that is, we do not actually reason in the Fourth, but only in the First, and then if occasion requires, convert the conclusion of the

First." But unless we quantify the predicate this account of the Fourth Figure cannot be accepted, since it will not apply to *Fesapo* or *Fresison*. For example, the premisses of *Fesapo* are,—

$$\text{No } P \text{ is } M,$$
$$\text{All } M \text{ is } S;$$

and, as they stand, we cannot obtain any conclusion whatever from them in Figure 1.

Thomson's ground of rejection is that "in the fourth figure the order of thought is wholly inverted, the subject of the conclusion had only been a predicate, whilst the predicate had been the leading subject in the premiss. Against this the mind rebels; and we can ascertain that the conclusion is only the converse of the real one, by proposing to ourselves similar sets of premisses, to which we shall always find ourselves supplying a conclusion so arranged that the syllogism is the first figure, with the second premiss first" (*Laws of Thought*, p. 178). With regard to the first part of this argument, Thomson himself points out that the same objection applies partially to Figures 2 and 3. It is no doubt a reason why as a matter of fact Figure 4 is seldom used; but I cannot see that it is a reason for altogether refusing to recognise it. The second part of Thomson's argument is, for a reason already stated, unsound. The conclusion, for example, of *Fresison* cannot be "the converse of the real conclusion," since (being an O proposition) it is the converse of nothing at all.

For my own part, I do not see how we can treat the syllogism scientifically and completely without admitting Figure 4. In an *a priori* separation of figures according to the position of the terms in the premisses, it necessarily appears, and we find that valid reasoning may be made in

it.   It is not actually in frequent use, but reasoning may
sometimes not unnaturally fall into it ; for example,—

> None of the apostles were Greeks,
> Some Greeks are worthy of all honour,

therefore, Some worthy of all honour are not apostles.

173.   The moods of Figure 4 regarded as indirect
moods of Figure 1.

The earliest form in which the mnemonic verses ap-
peared was as follows :—

> *Barbara, Celarent, Darii, Ferio, Baralipton,*
> *Celantes, Dabitis, Fapesmo, Frisesomorum,*
> *Cesare, Camestres, Festino, Baroco, Darapti,*
> *Felapton, Disamis, Datisi, Bocardo, Ferison*[1].

Aristotle recognised only three figures: the first figure,
which he considered the type of all syllogisms and which
he called the perfect figure, the *Dictum de omni et nullo*
being directly applicable to it alone ; and the second and
third figures, which he called imperfect figures, it being
necessary to reduce them to Figure 1, in order to obtain a
test of their validity.

Before the fourth figure, however, was commonly recog-
nised as such, its moods were recognised in another form,
namely, as *indirect* moods of the first figure ; and the above
mnemonics,—*Baralipton, Celantes, Dabitis, Fapesmo, Frise-
somorum,*—represent these moods so regarded[2].

---

[1] First given by Petrus Hispanus, afterwards Pope John XXI., who
died in 1277.

[2] From the 14th to the 17th century the mnemonics found in works
on Logic usually give the moods of Figure 4 in this form, or else omit
them altogether.   Wallis (1687) recognises them in both forms, giving
two sets of mnemonics.

Mansel (*Aldrich*, p. 78) defines an *indirect* mood as "one in which we do not infer the immediate conclusion, but its converse."

Thus,—   All *M* is *P*,

                   All *S* is *M*,

yields the direct conclusion, All *S* is *P*, in *Barbara*,

or the indirect conclusion, Some *P* is *S*, in *Baralipton*.

Similarly *Celantes* corresponds to *Celarent*, and *Dabitis* to *Darii*.

I should however take exception to Mansel's definition, since in *Fapesmo* and *Frisesomorum* we have indirect moods to which there are no corresponding valid direct moods. In these we do not infer "the converse of the immediate conclusion" since there is no immediate conclusion. Mansel deals with these two moods very awkwardly. He says, "*Fapesmo* and *Frisesomorum* have negative minor premisses, and thus offend against a special rule of the first figure; but this is checked by a counter-balancing transgression. For by simply converting O, we alter the distribution of the terms, so as to avoid an illicit process." But surely we cannot counterbalance one violation of law by committing a second. The truth of course is that, in the first place, the special rules of Figure 1 as ordinarily given do not apply to the indirect moods; and in the second place, the conclusion O is not obtained by conversion at all.

The real distinction between direct and indirect moods is that the terms which are major and minor in the one become respectively minor and major in the other.

Taking *Ferio*, and making this inversion, we have no valid conclusion, therefore, *Ferio* has no corresponding indirect mood. Similarly, *Fapesmo* and *Frisesomorum*, (the

Fesapo and *Fresison* of Figure 4), have no corresponding direct moods.

It is a merely formal difference whether we recognise the five moods in question in this way, or as constituting a distinct figure; but I think that the latter alternative is far less likely to give rise to confusion.

We have also indirect moods in Figures 2 and 3, but they are merely reproductions of other of the direct moods, as noticed by Professor Fowler (*Deductive Logic,* p. 104). Thus, the premisses,—

No *P* is *M*,
All *S* is *M*,

may yield both the conclusions No *S* is *P*, and No *P* is *S*; and if we call the former the *direct* conclusion the latter will be the *indirect*, (the former being *Cesare*, and the latter *Camestres* with the premisses transposed).

Some *P* is *M*,
No *S* is *M*,

yields no direct conclusion, but it has an indirect conclusion Some *P* is not *S*. This however merely gives *Festino* over again.

We get a similar result, in all cases in which this conception is applied within the limits of Figures 2 and 3.

174. " Rejecting the fourth figure and the subaltern moods, we may say with Aristotle ; **A** is proved only in one figure and one mood, **E** in two figures and three moods, **I** in two figures and four moods, **O** in three figures and six moods. For this reason, **A** is declared by Aristotle to be the most difficult proposition to establish, and the easiest to overthrow; **O**, the reverse."

Discuss the fitness of these data to support the conclusion.

The above quotation is from Mansel's *Aldrich*, p. 92. I do not think that the reasoning contained in it is sound. The difficulty of proving any proposition depends rather on the difficulty or easiness of obtaining premisses. Given the premisses, it is not the more difficult to prove the conclusion because it can only be done in one figure and one mood, nor the easier because it can be done in three figures and six moods. For it must be remembered that the doctrine of Reduction has shewn us that the different moods in which **O**, for instance, can be established are equivalent. If we were limited to proving **O** in *Ferio*, we could do it just as easily.

The above reasoning would make **E** easier to establish than **A**, and **O** than **I**; but this is not really the case. If we can prove an **E** we can also prove an **A** by obversion; and similarly with **O** and **I**.

For other reasons it is of course obvious that universals are more easily overthrown, particulars more easily established. A reference to the subaltern moods indeed illustrates this. **I** can be proved in all cases in which **A** can be proved, and some others; similarly with **O** and **E**.

THE DIAGRAMMATIC REPRESENTATION OF SYLLOGISMS.

175. The application of the Eulerian diagrams to Syllogistic reasonings.

To illustrate the application of the Eulerian diagrams to syllogistic reasonings we may take a syllogism in *Barbara*,—

<div align="center">

All *M* is *P*,

All *S* is *M*,

therefore, All *S* is *P*.

</div>

We must first represent the premisses separately by means of the diagrams. They each yield two cases; thus,—

All M is P,   (i)   MP   (ii)   M / P

All S is M,   (a)   SM   (b)   S / M
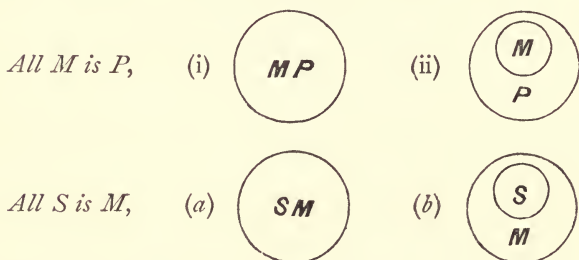
To obtain the conclusion, each of the cases yielded by the major premiss must now be combined with each of

those yielded by the minor. This gives four combinations, and whatever is true of *S* in terms of *P* in *all* of them, is the conclusion required.

(i) and (*a*) yield

(i) and (*b*)

(ii) and (*a*)

(ii) and (*b*)
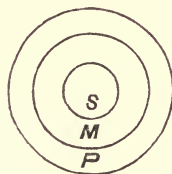
We find that in all these cases *all S is P*, which conclusion may therefore be inferred from the given premisses.

Next, take a syllogism in *Bocardo*. The application of the diagrams is now more complicated. The premisses are,—

> Some *M* is not *P*,
> All *M* is *S*.

The major premiss gives three possible cases,—

(i)   (ii)   (iii)
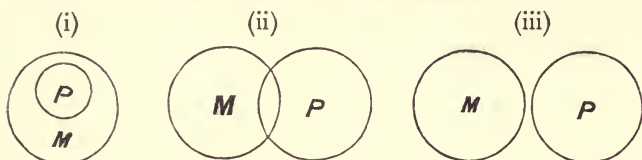
and the minor premiss gives two possible cases,—

(a)   (b)
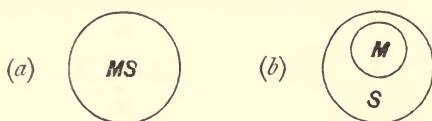
Taking them together we have six combinations, some of which however themselves yield more than one case:—

(i) and (a)

(i) and (b)

(ii) and (a)
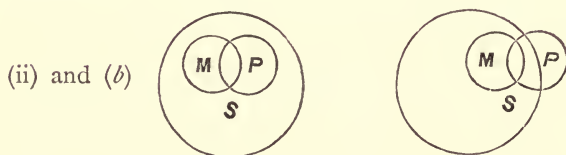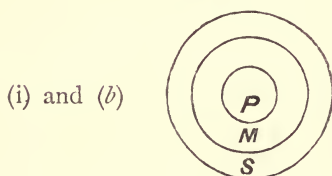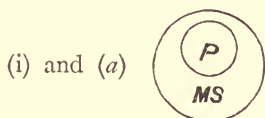
(ii) and (b)

(iii) and (*a*)

(iii) and (*b*)
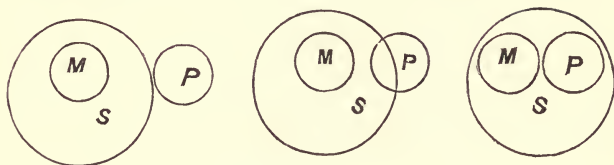
So far as *S* and *P* are concerned, (*i.e.*, leaving *M* out of account), it will be found that these nine cases are reducible to the following three,—
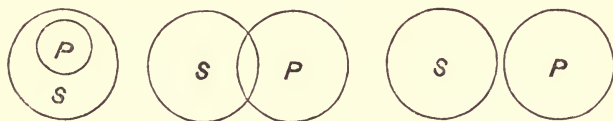
The conclusion therefore is *Some S is not P*.

It must be admitted that this is very complex, and it would be a serious matter if in the first instance we had to work through all the different moods in this manner. For purposes of illustration, however, this very complexity has a certain advantage. It shews how many relations between three terms in respect of extension are left to us, even with two premisses given.

176. Represent by means of the Eulerian diagrams the moods *Celarent, Festino, Datisi,* and *Bramantip.*

177. What is all that we can infer by the aid of the following premisses,—Some M is not P, Some M is P, Some P is not M, All M is S, Some S is not M?

The first three premisses limit us to,—
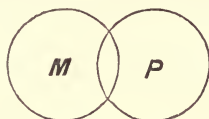
and the two remaining ones to,—
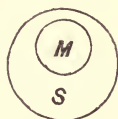
Combining these, we have,—
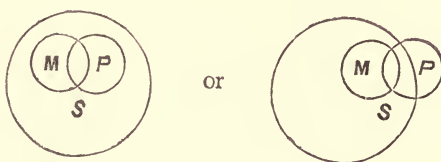
that is, so far as S and P are concerned, the premisses limit us to the following,—

These may be interpreted—

Some S is P, Some S is not P, and *Some P is S.*

We could of course obtain the same conclusions by the aid of the ordinary syllogistic moods.

178. The application of Mr Venn's diagrammatic scheme to syllogistic reasonings.

We may take *Barbara* and *Camestres* to illustrate the above.

The premisses of *Barbara*,—

<div style="text-align:center">

All M is P,

All S is M,

</div>

exclude certain compartments as shewn in the following diagram,—
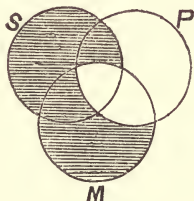
Then, so far as S and P are concerned, this is read off,— *All S is P*.

Similarly for *Camestres* we have,—
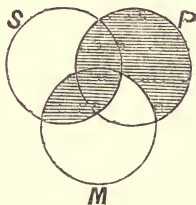
This scheme is especially adapted to illustrate the syllogistic processes when all the propositions involved are universal. A further device must be introduced when one of the premisses is particular. Compare section 95.

179. Represent *Celarent, Cesare, Camenes,* in Mr Venn's diagrammatic scheme.

180. Lambert's scheme of diagrammatic notation.

In the system of Lambert, (slightly altered so far as particular propositions are concerned, compare Venn, *Symbolic Logic*, p. 431), propositions are represented as follows:—

$$P$$
All S is P   ......———......
$$S$$

$$P$$
No S is P   ———
$$S$$

$$P$$
Some S is P   ......———......
$$S$$

Some S is not P   ......———......
$$P$$
$$S$$

It will be observed that the extension of a term is represented by a horizontal straight line, and that so far as two such lines overlap each other the corresponding classes are coincident, while so far as they do not overlap the corresponding classes exclude each other. The line is dotted in so far as we are uncertain with regard to a portion of the class; *i.e.*, a line representing an undistributed term is partly dotted. Thus, in the case of *All S is P*,—

$$P$$
......———......
$$S$$
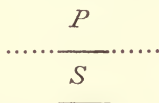
the diagram indicates that all *S* is contained under *P*, but that we are uncertain as to whether there is or is not any *P* which is not *S*.

In the case of *Some S is not P,*—

$$\begin{array}{c} P \\ \hline \\ S \quad \overline{} \\ \hline \text{\dots\dots\dots} \end{array}$$
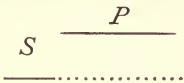
the diagram indicates that there is S which is not P, but that we are in ignorance as to the existence of any S that is P.

181. The application of Lambert's diagrammatic scheme to syllogistic reasonings.

As applied to syllogisms, the method indicated in the preceding section is much less cumbrous than the Eulerian diagrams[1]. We may take the following examples :—

Barbara

Baroco

---
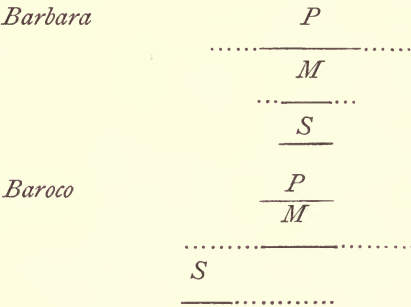
[1] Mr Venn (*Symbolic Logic*, p. 432) remarks,—"As a whole Lambert's scheme seems to me distinctly inferior to the scheme of Euler, and has in consequence been very little employed by other logicians." Mr Venn's criticism is chiefly directed against Lambert's representation of the particular affirmative proposition, namely,—

$$\begin{array}{c} P \quad\text{\rule{2cm}{0.4pt}} \\ S \text{\dots\dots\dots} \end{array}$$
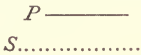
The modification, however, which I have here introduced, and which is suggested by Mr Venn himself, meets the objections raised on this ground.

Datisi
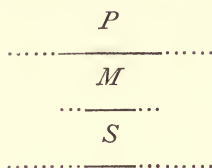
Fresison
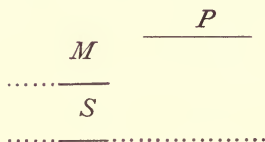
182. Represent the moods *Darii, Cesare, Darapti,* and *Fesapo* in Lambert's scheme.

183. Take the premisses of an ordinary syllogism in *Barbara, e.g.,* all *X* is *Y*, all *Y* is *Z* ; determine precisely and exhaustively what those propositions affirm, what they deny, and what they leave in doubt, concerning the relations of the terms *X, Y, Z.* [L.]

This question can be very well answered by the aid of any of the three diagrammatic schemes which we have just been discussing. Compare also Jevons, *Studies in Deductive Logic,* p. 216.

CHAPTER VI.

IRREGULAR AND COMPOUND SYLLOGISMS.

184. The Enthymeme.

By the *Enthymeme*, Aristotle meant what has been called the "rhetorical syllogism" as opposed to the apodeictic, demonstrative, theoretical syllogism. The following is from Mansel's notes to *Aldrich* (pp. 209—211): "The Enthymeme is defined by Aristotle, συλλογισμὸς ἐξ εἰκότων ἤ σημείων. The εἰκὸς and σημεῖον themselves are Propositions; the former stating a *general probability*, the latter a *fact*, which is known to be an indication, more or less certain, of the truth of some further statement, whether of a single fact or of a general belief. The former is a proposition nearly, though not quite, *universal;* as 'Most men who envy hate': the latter is a *singular* proposition, which however is not regarded as a sign, except relatively to some other proposition, which it is supposed may be inferred from it. The εἰκός, when employed in an Enthymeme, will form the *major premiss* of a Syllogism such as the following:

> Most men who envy hate,
>
> This man envies,

therefore,   This man (probably) hates.

14—2

The reasoning is logically faulty; for, the major premiss not being absolutely universal, the middle term is not distributed.

The σημεῖον will form one premiss of a Syllogism which may be in any of the three figures, as in the following examples :

Figure 1.   All ambitious men are liberal,
                Pittacus is ambitious,
    Therefore, Pittacus is liberal.

Figure 2.   All ambitious men are liberal,
                Pittacus is liberal,
    Therefore, Pittacus is ambitious.

Figure 3.   Pittacus is liberal,
                Pittacus is ambitious,
    Therefore, All ambitious men are liberal.

The syllogism in the first figure is alone logically valid. In the second, there is an undistributed middle term : in the third, an illicit process of the minor."

On this subject the student may be referred to the remainder of the note from which the above extract is taken, and to Hamilton, *Discussions*, pp. 152—156.

An *enthymeme* is now usually defined as a syllogism incompletely stated, one of the premisses or the conclusion being understood but not expressed. As has been frequently pointed out, the arguments of everyday life are for the most part enthymematic. The same may be said of fallacious arguments, which are seldom completely stated, or their want of cogency would be more quickly recognised.

An enthymeme is said to be of the first order when the major premiss is suppressed ; of the second order when the minor premiss is suppressed ; and of the third order when the conclusion is suppressed.

Thus, " Balbus is avaricious, and therefore, he is un-happy," is an enthymeme of the first order ; " All avaricious persons are unhappy, and therefore, Balbus is unhappy " is an enthymeme of the second order; "All avaricious persons are unhappy, and Balbus is avaricious " is an enthymeme of the third order.

185. The Polysyllogism ; and the Epicheirema.

A chain of syllogisms, that is, a series of syllogisms so linked together that the conclusion of one becomes a pre-miss of another, is called a *polysyllogism.* In a polysyllogism, any individual syllogism the conclusion of which becomes the premiss of a succeeding one is called a *prosyllogism;* any individual syllogism one of the premisses of which is the conclusion of a preceding syllogism is called an *epi-syllogism.* Thus,—

$$
\left.
\begin{array}{l}
\text{All } C \text{ is } D, \\
\text{All } B \text{ is } C,
\end{array}
\right\} \text{ prosyllogism,}
$$
$$
\left.
\begin{array}{l}
\text{therefore, All } B \text{ is } D, \\
\text{but, All } A \text{ is } B, \\
\text{therefore, All } A \text{ is } D.
\end{array}
\right\} \text{ episyllogism.}
$$

The same syllogism may of course be both an episyllo-gism and a prosyllogism, as would be the case with the above episyllogism if the chain were continued further.

An *epicheirema* is a polysyllogism with one or more prosyllogisms briefly indicated only. That is, one or more of the syllogisms of which the polysyllogism is composed is enthymematic. Whately (*Logic,* p. 117) calls it accord-ingly an *enthymematic sentence.* The following is an example,

$$B \text{ is } D \text{, because it is } C,$$
$$A \text{ is } B,$$
$$\text{therefore, } A \text{ is } D.$$

186. The Sorites.

A *Sorites* is a polysyllogism in which all the conclusions are omitted except the final one; for example,

$$A \text{ is } B,$$
$$B \text{ is } C,$$
$$C \text{ is } D,$$
$$D \text{ is } E,$$
$$\text{therefore, } A \text{ is } E.$$

187. The ordinary Sorites, and the Goclenian Sorites.

In the ordinary Sorites, the premiss which contains the subject of the conclusion is stated first; in the Goclenian Sorites it is stated last. Thus,—

Ordinary Sorites,— A is B,
$$B \text{ is } C,$$
$$C \text{ is } D,$$
$$D \text{ is } E,$$
$$\text{therefore, } A \text{ is } E.$$

Goclenian Sorites,— D is E,
$$C \text{ is } D,$$
$$B \text{ is } C,$$
$$A \text{ is } B,$$
$$\text{therefore, } A \text{ is } E.$$

If, in the case of the ordinary sorites, the argument were drawn out in full, the suppressed conclusions would appear as minor premisses in successive syllogisms. Thus, the ordinary sorites given above may be analysed into the three following syllogisms,—

(1) B is C,
$$A \text{ is } B,$$
$$\text{therefore, } A \text{ is } C;$$

(2) C is D,
                               A is C,
              therefore, A is D;

(3) D is E,
                               A is D,
              therefore, A is E.

Here the suppressed conclusion of (1) is seen to be the minor premiss of (2), that of (2) the minor premiss of (3); and so it would go on if the number of propositions constituting the Sorites were increased.

In the Goclenian Sorites, the premisses are the same, but their order is reversed, and the result of this is that the suppressed conclusions become *major* premisses in successive syllogisms.

Thus the Sorites,— D is E,
                       C is D,
                       B is C,
                       A is B,
        therefore, A is E,

may be analysed into the following three syllogisms,—

(1) D is E,
                               C is D,
              therefore, C is E;

(2) C is E,
                               B is C,
              therefore, B is E;

(3) B is E,
                               A is B,
              therefore, A is E.

Here the conclusion of (1) becomes the major premiss of (2); and so on.

The ordinary Sorites[1] is that which is most usually discussed; but it may be noted that the order of premisses in the Goclenian form is that which really corresponds to the customary order of premisses in a simple syllogism.

188. The special rules of the ordinary Sorites.

The special rules of the ordinary sorites are,—

(1) Only one premiss can be negative; and if one is negative, it must be the last.

(2) Only one premiss can be particular; and if one is particular, it must be the first.

Any ordinary sorites may be represented in skeleton form, the quantity and quality of the premisses being left undetermined, as follows :—

---

[1] What I have called the ordinary Sorites is frequently spoken of as the *Aristotelian* Sorites; for example, by Archbishop Thomson (*Laws of Thought*, p. 201), and Spalding (*Logic*, p. 302). Hamilton however remarks,—" The name Sorites does not occur in any logical treatise of Aristotle; nor, as far as I have been able to discover, is there, except in one vague and cursory allusion, any reference to what the name is now employed to express" (*Lectures on Logic*, I. p. 375). The term Sorites (from σωρὸς, a heap) as used by ancient writers was employed to designate a particular sophism, based on the difficulty which is sometimes found in assigning an exact limit to a notion. "It was asked,—was a man bald who had so many thousand hairs; you answer, No : the antagonist goes on diminishing and diminishing the number, till either you admit that he who was not bald with a certain number of hairs, becomes bald when that complement is diminished by a single hair; or you go on denying him to be bald, until his head be hypothetically denuded."

The distinct exposition of the kind of reasoning which is now known as the Sorites is attributed to the Stoics; but it was not called by this name till the fifteenth century (Hamilton, *Logic*, I. p. 377). The form of Sorites called the Goclenian was "first given by Goclenius in his *Isagoge in Organum Aristotelis*, 1598" (Mansel's *Aldrich*, p. 96).

CHAP. VI.]   SYLLOGISMS.   217

$$
\begin{array}{cc}
S & M_1 \\
M_1 & M_2 \\
M_2 & M_3 \\
\cdots\cdots\cdots \\
\cdots\cdots\cdots \\
M_{n-2} & M_{n-1} \\
M_{n-1} & M_n \\
M_n & P \\
\hline
S & P
\end{array}
$$

(1) There cannot be more than one negative premiss, for if there were, (since a negative premiss in any syllogism necessitates a negative conclusion), we should in analysing the sorites somewhere come upon a syllogism containing two negative premisses.

Again, if one premiss is negative, the final conclusion must be negative. Therefore, P must be distributed in this conclusion. Therefore, it must be distributed in its premiss, *i.e.*, the last premiss, which must therefore be negative.

If any premiss then is negative, this is the one.

(2) Since it has been shewn that all the premisses, except the last, must be affirmative, it is clear that if any, except the first, were particular, we should somewhere commit the fallacy of undistributed middle.

189. Find and prove the special rules of the Goclenian Sorites.

190. The possibility of a Sorites in a Figure other than the First.

It will have been noticed that in analysing both the (so-called) Aristotelian and the Goclenian Sorites all the resultant syllogisms are in Figure 1. Such sorites therefore may themselves be said to be in Figure 1. The question arises whether a sorites is possible in any other figure.

Sir William Hamilton (*Lectures on Logic*, vol. 2, p. 403) remarks that "all logicians have overlooked the Sorites of Second and Third Figures." Reading on, however, we find that by a Sorites in the Second Figure he means such a reasoning as the following:—No B is A, No C is A, No D is A, No E is A, All F is A, therefore, No B, or C, or D or E, is F; and by a Sorites in the Third Figure such as the following:—A is B, A is C, A is D, A is E, A is F, therefore, Some B, and C, and D, and E, are F. (He does not himself give these examples; but that this is what he means may be deduced from his not very lucid statement,—" In Second and Third Figures, there being no subordination of terms, the only Sorites competent is that by repetition of the same middle. In First Figure, there is a new middle term for every new progress of the Sorites; in Second and Third, only one middle term for any number of extremes. In First Figure, a syllogism only between every second term of the Sorites, the intermediate term constituting the middle term. In the others, every two propositions of the common middle term form a syllogism.")

But it is clear that in the accepted sense of the term these are not sorites at all. In neither of them have we any chain argument, but our conclusion is a mere summation of the conclusions of a number of syllogisms having a common premiss.

Hamilton's own definition of sorites, involved as it is, might have saved him from this error. He gives for his definition,—" When, on the common principle of all reasoning,—that the part of a part is a part of the whole,—we do not stop at the second gradation, or at the part of the highest part, and conclude that part of the whole, but proceed to some indefinitely remoter part, as D, E, F, G, H, &c., which, on the general principle, we connect in the conclusion

with its remotest whole,—this complex reasoning is called
a *Chain-Syllogism* or *Sorites*" (*Lectures on Logic*, vol. I.
p. 366).

In the above criticism I have followed J. S. Mill[1]. His
own treatment of the question, however, seems open to
refutation by the simple method of constructing examples.
He considers that the first or last syllogism of a sorites
may be in Figure 2 or 3, (*e.g.*, in Figure 2 we might have
A is *B*, *B* is *C*, *C* is *D*, *D* is *E*, No *F* is *E*, therefore *A* is
not *F*), but that it is impossible that all the steps should be
in either of these figures. " Every one who understands the
laws of the second and third figures (or even the general laws
of the syllogism) can see that no more than one step in
either of them is admissible in a sorites, and that it must
either be the first or the last."

But take the following (the suppressed conclusions being
inserted in square brackets):—

<div style="text-align:center">

All A is B,

No C is B,

[therefore, No *A* is *C*],

All D is C,

[therefore, No *A* is *D*],

All E is D,

[therefore, No *A* is *E*],

All F is E,

therefore, *No A is F*[2].

</div>

---

[1] In connection with it, Mill very justly remarks,—" If Sir W.
Hamilton had found in any other writer such a misuse of logical
language as he is here guilty of, he would have roundly accused him of
total ignorance of logical writers" (*Examination of Hamilton*, p. 515).

[2] This Sorites is analogous to the so-called Aristotelian Sorites,
the subject of the conclusion appearing in the premiss stated first. It is
to be observed that the rules given in section 188 will not apply except

All the syllogisms involved here are in Figure 2, and the sorites itself may I think fairly be said to be in Figure 2. As in the ordinary sorites, the conclusion of each syllogism is the minor of the next.

The following again may be called a sorites in Figure 3:—

<div align="center">

All B is A,

All B is C,

[therefore, Some *C* is *A*],

All C is D,

[therefore, Some *D* is *A*],

All D is E,

therefore, *Some E is A*,

therefore, *Some A is E*[1].

</div>

Here the conclusion of each syllogism is the major of the next[2].

191. Take any Enthymeme (in the modern sense) and supply premisses so as to expand it into (*a*) a syllogism, (*b*) an epicheirema, (*c*) a sorites; and name the mood, order or variety of each product.   [C.]

192. Is there any case in which a conclusion can be obtained from two premisses, although the middle term is distributed in neither of them?

The ordinary syllogistic rule relating to the distribution of the middle term does not contemplate the recognition of

when the Sorites is in Figure 1. For Sorites in Figures 2 and 3, however, other rules might be framed corresponding to the special rules of Figures 2 and 3 in the case of the simple Syllogism.

[1] The preceding note applies to this Sorites also.

[2] I should admit that such Sorites as the above are not likely to be found in common use.

any signs of quantity other than *all* and *some*. The admission of the sign *most* yields the valid reasoning,—

<div align="center">

Most M is P,

Most M is S,

therefore, *Some S is P*.

</div>

We understand *most* in the sense of *more than half*, and it clearly follows from the above premisses that there must be some M which is both S and P. We cannot however say that in either premiss the term M is distributed. To meet this case, then, the rule with regard to the distribution of the middle term must be amended, if other signs of quantity besides *all* and *some* are recognised.

Sir W. Hamilton (*Logic*, vol. 2, p. 362) gives,—"The quantifications of the middle term, whether as subject or predicate, taken together, must exceed the quantity of that term taken in its whole extent"; in other words, we require the *ultra-total* distribution of the middle term, in the two premisses taken together. Hamilton then continues somewhat too dogmatically,—"The rule of the logicians, that the middle term should be once at least distributed, is untrue. For it is sufficient if, in both the premisses together, its quantification be more than its quantity as a whole, (ultratotal). Therefore, a *major part*, (a *more* or *most*), in one premiss, and a *half* in the other, are sufficient to make it effective."

De Morgan (*Formal Logic*, p. 127) writes as follows,—"It is said that in every syllogism the middle term must be universal in one of the premisses, in order that we may be sure that the affirmation or denial in the other premiss may be made of some or all of the things about which affirmation or denial has been made in the first. This law, as we shall see, is only a particular case of the truth: it

is enough that the two premisses together affirm or deny of more than all the instances of the middle term. If there be a hundred boxes, into which a hundred *and one* articles of two different kinds are to be put, not more than one of each kind into any one box, some one box, if not more, will have two articles, one of each kind, put into it. The common doctrine has it, that an article of one particular kind must be put into every box, and then some one or more of another kind into one or more of the boxes, before it may be affirmed that one or more of different kinds are found together." De Morgan himself works the question out in detail in his treatment of *the numerically definite syllogism*, (*Formal Logic*, pp. 141—170).

193. The Argument *a fortiori* and other deductive inferences that are not reducible to the ordinary syllogistic form.

We may take as an example of the argument *a fortiori:*

<p style="text-align:center">B is greater than C,</p>
<p style="text-align:center">A is greater than B,</p>

therefore, A is greater than C.

As this stands, it is clearly not in the ordinary syllogistic form since it contains four terms; some logicians however profess to reduce it to the ordinary syllogistic form as follows:

<p style="text-align:center">B is greater than C,</p>

therefore, a greater than B is greater than C,

but, A is a greater than B,

therefore, A is greater than C.

With De Morgan, we may treat this as a mere evasion, or as a *petitio principii*. The principle of the argument *a fortiori* is really assumed in passing from "B is greater than C" to "a greater than B is greater than C."

The following attempted resolution[1] must I think be disposed of similarly :

Whatever is greater than a greater than C is greater than C,

         A is greater than a greater than C,

therefore, A is greater than C.

At any rate, it is clear that this cannot be the whole of the reasoning, since B no longer appears in the premisses at all.

Mansel (*Aldrich*, pp. 199, 200) treats the argument *a fortiori* as a *material consequence*, and by this he means, "one in which the conclusion follows from the premisses solely by the force of the terms," *i.e.*, "from some understood proposition or propositions, connecting the terms, by the addition of which the mind is enabled to reduce the consequence to logical form." He would reduce the argument *a fortiori* in one of the ways already referred to. This however begs the question that the syllogistic is the only *logical* form. As a matter of fact the cogency of the argument *a fortiori* is just as intuitively evident as that of a syllogism in *Barbara* itself. Why should no relation be regarded as *formal* unless it can be expressed by the word *is*? Touching on this case, De Morgan remarks that the formal logician has a right to confine himself to any part of his subject that he pleases; "but he has no right except the right of fallacy to call that part the whole" (*Syllabus*, p. 42).

"A equals B; B equals C; therefore, A equals C" is another case to which the same remarks apply.

"This is not an instance of common syllogism: the premisses are 'A *is* an equal of B; B *is* an equal of C.' So

<hr />

[1] Cf. Mansel's *Aldrich*, p. 200.

far as common syllogism is concerned, that 'an equal of B'
is as good for the argument as 'B' is a *material* accident of
the meaning of 'equal.' The logicians accordingly, to re-
duce this to a common syllogism, state the effect of com-
position of relation in a major premiss, and declare that
the case before them is an example of that composition in
a minor premiss. As in, A is *an equal of an equal* (of C);
Every *equal of an equal* is an *equal;* therefore, A is an *equal* of
C. This I treat as a mere evasion. Among various suffi-
cient answers this one is enough: *men do not think as
above.* When $A = B$, $B = C$, is made to give $A = C$, the
word *equals* is a *copula* in thought, and not a *notion attached
to a predicate.* There are processes which are not those of
common syllogism in the logician's major premiss above:
but waiving this, logic is an analysis of the form of thought,
possible and actual, and the logician has no right to declare
that other than the actual is actual." (De Morgan, *Syllabus,*
pp. 31, 2.)

There are an indefinite number of other arguments
which for similar reasons cannot be reduced to syllogistic
form. For example,—X is a contemporary of Y, and
Y of Z; therefore X is a contemporary of Z. A is the
brother of B, B is the brother of C; therefore, A is the
brother of C.

We must then reject the claims that have been put for-
ward on behalf of the syllogism to be the exclusive form of
all deductive reasoning.

As an example of such claims being made, Whately may
be quoted. Syllogism, he says, is "the form to which *all*
correct reasoning may be ultimately reduced" (*Logic,* p. 12).
Again, he remarks, "An argument thus stated regularly and
at full length, is called a Syllogism; which therefore is
evidently not a peculiar *kind of argument*, but only a peculiar

form of expression, in which every argument may be stated " (*Logic*, p. 26)[1].

Spalding seems to have the same thing in view when he says,—"An inference, whose antecedent is constituted by more propositions than one, is a Mediate Inference. The simplest case, that in which the antecedent propositions are two, is the Syllogism. The syllogism is the norm of all inferences whose antecedent is more complex ; and all such inferences may, by those who think it worth while, be resolved into a series of syllogisms" (*Logic*, p. 158).

J. S. Mill endorses these claims. He remarks,—"All valid ratiocination ; all reasoning by which from general propositions previously admitted, other propositions equally or less general are inferred ; may be exhibited in some of the above forms," *i.e.*, the syllogistic moods, (*Logic*, I. p. 191).

What is required to fill the logical gap which is created by the admission that the syllogism is *not* the norm of all valid formal inference has been called the *Logic of Relatives*. The function of the Logic of Relatives is to "take account of relations generally, instead of those merely which are indicated by the ordinary logical copula *is* ", (Venn, *Symbolic Logic*, p. 400). The line which this new Logic is likely to take, if it is ever fully worked out, is indicated by the following passage from De Morgan (*Syllabus*, pp. 30, 31):—

"A *convertible* copula is one in which the copular relation exists between two names *both ways :* thus 'is fastened to,' 'is joined by a road with,' 'is equal to,' 'is in habit of conversation with,' &c. are *convertible* copulæ. If '*X* is equal to *Y*' then '*Y* is equal to *X*,' &c. A *transitive* copula is one in which the copular relation joins *X* with *Z* whenever it

---

[1] Cf. also Whately, *Logic*, pp. 24, 5, and p. 34.

joins X with Y and Y with Z. Thus 'is fastened to' is usually understood as a transitive copula : 'X is fastened to Y' and 'Y is fastened to Z' give 'X is fastened to Z.' All the copulæ used in this syllabus are *transitive*. The intransitive copula cannot be treated without more extensive consideration of the combination of relations than I have now opportunity to give : a second part of this syllabus or an augmented edition, may contain something on this subject." The Student may further be referred to Venn, *Symbolic Logic*, pp. 399—404.

CHAPTER VII.

HYPOTHETICAL SYLLOGISMS.

194. The Hypothetical Syllogism and the Hypothetico-Categorical Syllogism.

The form of reasoning in which a hypothetical conclusion is inferred from two hypothetical premisses is apparently overlooked by some logicians; at any rate, it frequently receives no distinct recognition, the term "hypothetical syllogism" being limited to the case in which one premiss only is hypothetical.

I should however prefer the following definitions :—

A *Hypothetical Syllogism* is a mediate reasoning consisting of three propositions in which both the premisses and the conclusion are hypothetical in form;

> *e. g.,—If C is D, E is F,*
> *If A is B, C is D,*
> therefore, *If A is B, E is F.*

A *Hypothetico-Categorical Syllogism* is a mediate reasoning consisting of three propositions in which one of the premisses is hypothetical in form, while the other premiss and the conclusion are categorical;

> *e. g.,—If A is B, C is D,*
> *A is B,*
> therefore, *C is D.*

This nomenclature is adopted by Spalding and Ueber-weg, but, as I have already hinted, it is not the most usual. Some logicians, (*e. g.*, Fowler), call either of the above forms of reasoning hypothetical syllogisms without distinction. Others, (*e. g.*, Jevons), define the hypothetical syllogism so as to include the latter form alone, the former apparently not being regarded by them as a distinct form of reasoning at all. This view may be to some extent justified by the very close analogy that exists between the syllogism with two hypothetical premisses and the categorical syllogism; but the difference in form is worth at least a brief discussion.

The student should however bear in mind that by the "hypothetical syllogism" in most English works on Logic is meant what has been defined above as the hypothetico-categorical syllogism.

195. Distinction of Figure and Mood in the case of Hypothetical Syllogisms.

In the Hypothetical Syllogism, (as defined in the preceding section), the antecedent of the conclusion is equivalent to the minor term of the categorical syllogism, the consequent of the conclusion to the major term, and the element which does not appear in the conclusion at all to the middle term. Distinctions of mood and figure may be recognised in precisely the same way as in the case of the categorical syllogism. For example,—

> *Barbara,*—*If C is D, E is F,*
> *If A is B, C is D,*
> therefore, *If A is B, E is F.*

> *Festino,*— *If E is F, C is not D.*
> *In some cases in which A is B, C is D,*
> therefore, *In some cases in which A is B, E is not F.*

Darapti,—If C is D, E is F,
If C is D, A is B,
therefore, *In some cases in which A is B, E is F.*

Camenes,—If E is F, C is D,
If C is D, A is not B,
therefore, *If A is B, E is not F.*

In working with hypotheticals it must always be remembered that the quality of the proposition is determined by the quality of the consequent.

196. The Reduction of Hypothetical Syllogisms.

Hypothetical Syllogisms in Figures 2, 3, 4 may be reduced to Figure 1 just as in the case of Categorical Syllogisms. Thus, the syllogism in *Camenes* given in the preceding example is reduced as follows to Camestres,—

If C is D, A is not B,
If E is F, C is D,
therefore, *If E is F, A is not B,*
therefore, *If A is B, E is not F.*

According to rule, the premisses have here been transposed, and the conclusion of the new syllogism is converted in order to obtain the original conclusion.

197. Construct Hypothetical Syllogisms in *Cesare, Bocardo, Fesapo,* and reduce them to Figure 1.

198. Name the mood and figure of the following:

(1) *If C is D, E is not F,*
In some cases in which A is B, C is D,
therefore, *In some cases in which A is B, E is not F.*

(2)   *If E is F, C is D,*
       If C is D, A is B,
therefore, *In some cases in which A is B, E is F.*

Shew that one of these forms may be indirectly reduced to the other, but not *vice versa*. Why is this?

199.   Name the mood and figure of the following, and shew that either one may be reduced to the other form :—

(1)   *If E is not F, C is D,*
       If A is B, C is not D,
therefore, *If A is B, E is F.*

(2)   *If C is D, E is not F,*
       If A is not B, C is D,
therefore, *If A is not B, E is not F.*

200.   The Moods of the Hypothetico-categorical Syllogism.

It is usual to distinguish two moods of the hypothetico-categorical syllogism:

(1)   The *modus ponens*, (also called the *constructive* hypothetical syllogism), in which the categorical premiss affirms the antecedent of the hypothetical premiss, thereby justifying as a conclusion the affirmation of its consequent.   For example,—

If A is B, A is C,
       A is B,
therefore,   *A is C.*

(2)   The *modus tollens*, (also called the *destructive* hypothetical syllogism), in which the categorical premiss denies

the consequent of the hypothetical premiss, thereby justify-
ing as a conclusion the denial of its antecedent. For ex-
ample,—

$$\textit{If A is B, A is C,}$$
$$\textit{A is not C,}$$
$$\text{therefore, } \textit{A is not B.}$$

These may be considered to correspond respectively to
Figures 1 and 2 of the categorical syllogism.

Thus, the example of *modus ponens* given above may be
written,—

$$\textit{All cases of A being B are cases of A being C,}$$
$$\textit{This case of A is a case of A being B,}$$
$$\text{therefore, } \textit{This case of A is a case of A being C;}$$

and we then have a syllogism in *Barbara*.

The following corresponds to *Celarent*,—

$$\textit{If A is B, A is not C,}$$
$$\textit{A is B,}$$
$$\text{therefore, } \textit{A is not C.}$$

The example of *modus tollens* given above corresponds
to *Camestres*. The following corresponds to *Cesare*,—

$$\textit{If A is B, A is not C,}$$
$$\textit{A is C,}$$
$$\text{therefore, } \textit{A is not B.}$$

201. Reduction of the *modus tollens* to the *modus
ponens*.

Any case of *modus tollens* may be reduced to *modus
ponens* and *vice versa*.

Thus, $$\textit{If A is B, A is C,}$$
$$\textit{A is not C,}$$
$$\text{therefore, } \textit{A is not B,}$$

becomes by contraposition of the hypothetical premiss,

$$\textit{If A is not C, A is not B,}$$
$$\textit{A is not C,}$$
therefore,   *A is not B ;*

and this is *modus ponens.*

It may be worth noticing here that a categorical syllogism in *Camestres* may similarly be reduced to *Celarent* without transposing the premisses:—

$$\textit{All P is M,}$$
$$\textit{No S is M,}$$
therefore,   *No S is P.*

$$\textit{No not-M is P,}$$
$$\textit{All S is not-M,}$$
therefore,   *No S is P.*

202. Shew how the *modus ponens* may be reduced to the *modus tollens.*

203. Mention two fallacious modes of arguing from a hypothetical major premiss. To what fallacies in categorical syllogisms do they respectively correspond ?   [C.]

There are two principal fallacies to which we are liable in arguing from a hypothetical major premiss:—

(1) It is a fallacy if we regard the affirmation of the consequent as justifying the affirmation of the antecedent. For example,

$$\textit{If A is B, A is C,}$$
$$\textit{A is C,}$$
therefore,   *A is B* [1].

---

[1] This would of course be no longer a fallacy if *A is B* were given as the *sole* condition of *A is C.*

(2) It is a fallacy if we regard the denial of the antecedent as justifying the denial of the consequent. For example,

$$If\ A\ is\ B,\ A\ is\ C,$$
$$A\ is\ not\ B,$$
$$therefore,\ A\ is\ not\ C\,[1].$$

It will easily be seen that these correspond respectively to *undistributed middle* and *illicit major* in the case of categorical syllogisms.

204. The claims of the Hypothetico-categorical Syllogism to be regarded as Mediate Inference.

Taking the syllogism,—
$$If\ A\ is\ B,\ C\ is\ D,$$
$$but\ A\ is\ B,$$
$$therefore,\ C\ is\ D,$$

the conclusion is at any rate apparently obtained by a combination of two premisses, and the burden of proof certainly seems to lie with those who deny the claims of such an inference as this to be called mediate inference.

Professor Bain's arguments, (*Logic, Deduction,* p. 117), upon this point are not easy to formulate; but they resolve themselves into one or other or both of the following:—

(1) He seems to argue that the so-called hypothetical syllogism is not really mediate inference, *because* it is "a pure instance of the Law of Consistency"; in other words, because "the conclusion is implied in what has already been stated." But is not this the case in all formal mediate inference? Professor Bain cannot consistently maintain that the categorical syllogism is more than a pure instance

<hr>

[1] See note on the preceding page.

of the Law of Consistency; or that the conclusion in such a syllogism is not implied in what has already been stated.

(2) But he may mean that the conclusion is implied in the hypothetical premiss alone. Indeed he goes on to say, "'If the weather continues fine, we shall go into the country' is transformable into the equivalent form 'The weather continues fine, and so we shall go into the country.' Any person affirming the one, does not, in affirming the other, declare a new fact, but the same fact." If this is intended to be understood literally, it is to me a very extraordinary statement. Take the following :—If a Russian army lands in Britain, the volunteers will be called out; If the sun moves round the earth, modern astronomy is utterly wrong. Are these respectively equivalent to,—the Russians have landed in Britain and so the volunteers are being called out; the sun moves round the earth, and so modern astronomy is utterly wrong? Besides, if the proposition *If A is B, C is D* implies that *A is B*, what becomes of the possible reasoning, "But *C is not D*, therefore, *A is not B*"?

Further arguments in favour of Bain's view are as follows :—

(1) "There is no middle term in the so-called hypothetical syllogism." The answer is that there is something in the premisses which does not appear in the conclusion, and that this corresponds to the middle term of the categorical syllogism. If we reduce the hypothetical syllogism to the categorical form, this is more distinctly recognisable.

(2) "In the so-called hypothetical syllogism, the minor and the conclusion indifferently change places." This statement is erroneous. Taking the syllogism stated at the commencement of this section and transposing the so-called minor and the conclusion, we have a fallacy. Compare section 203.

(3) "The major in a so-called hypothetical syllogism consists of two propositions, the categorical major of two terms." This merely tells us that a hypothetical syllogism is not the same in form as a categorical syllogism, but seems to have no bearing on the question whether the so-called hypothetical syllogism is a case of mediate or of immediate inference.

Turning now to the other side of the question, I do not see what satisfactory answers can be given to the following arguments in favour of regarding the hypothetico-categorical syllogism as a case of mediate inference. In any such syllogism, the two premisses are quite distinct, neither can be inferred from the other, but both are necessary in order that the conclusion may be obtained. Again, if we compare with it the inferences which are on all sides admitted to be immediate inferences from the hypothetical proposition, the difference between the two cases is apparent. From *If A is B, C is D* I can infer immediately *If C is not D, A is not B*; but I require also to know that *C is not D* in order to be able to infer that *A is not B*.

It has also been shewn that a reasoning which naturally falls into the form of the hypothetico-categorical syllogism may nevertheless be exhibited in the form of the ordinary categorical syllogism, which is admitted to be a case of mediate reasoning. Moreover there are distinct forms,— the *modus ponens* and the *modus tollens*,—which correspond to distinct forms of the categorical syllogism ; and fallacies in the hypothetical syllogism correspond exactly to certain fallacies in the categorical syllogism.

Professor Bowen indeed remarks (*Logic*, p. 265):—"The reduction of a Hypothetical Judgment to a Categorical shews very clearly the Immediacy of the reasoning in what is called a Hypothetical Syllogism. Thus, *If A is B, C is D*,

is equivalent to All cases of A is B are cases of C is D, therefore,

$$\left\{\begin{array}{l}\text{Some cases of } A \text{ is } B \text{ are cases of}\\ \text{This case of } A \text{ is } B \text{ is a case of}\end{array}\right\} C \text{ is } D.\text{"}$$

But does not this overlook the fact that a new judgment is required to tell me that this *is* a case of A is B? The mere statement that some cases of A is B are cases of C is D is clearly not equivalent to the conclusion of the hypothetical syllogism.

In the case of the *modus tollens*,—" If A is B, C is D; but C is not D; therefore, A is not B",—the argument in favour of regarding it as mediate inference is still more forcible; but of course the *modus ponens* and the *modus tollens* stand and fall together[1].

Professor Croom Robertson (*Mind*, 1877, p. 264) has suggested an explanation as to the manner in which this controversy may have arisen. He distinguishes the *hypothetical* "if" from the *inferential* "if," the latter being equivalent to *since, seeing that, because*. No doubt by the aid of a certain accentuation the word "if" may be made to carry with it this force. Professor Robertson quotes a passage from *Clarissa Harlowe* in which the remark "If you have the value for my cousin that you say you have, you must needs think her worthy to be your wife," is explained by the speaker to mean, "*Since* you have, &c." Using the word in this sense, the conclusion "C is D" certainly follows immediately from the bare statement, "If A is B, C is D"; or rather this statement itself affirms the conclusion. We cannot however regard the word "if" as *logically* carrying with it this inferential implication. When it is so used we

---

[1] In section 210 I shew further that the Hypothetical Syllogism and the Disjunctive Syllogism also stand and fall together.

have really a condensed mode of expression including two statements in one; I should indeed turn the argument the other way by saying that in the single statement thus interpreted we have a hypothetical syllogism expressed elliptically[1].

205. If A is true, B is true; if B is true, C is true; if C is true, D is true. What is the effect upon the other assertions of supposing successively (1) that D is false; (2) that C is false; (3) that B is false; (4) that A is false? [Jevons, *Studies*, p. 146.]

206. Examine the following:

If none but B are A, it cannot be possible that any X are Y; but all X are Y; therefore Some A are not B.

If the reasoning is correct, reduce it to proper syllogistic mood and figure. [v.]

207. Let X, Y, Z, P, Q, R, be six propositions: given, (*a*) Of X, Y, Z, one and only one is true;

(*b*) Of P, Q, R, one and only one is true;

(*c*) If X is true, P is true;

(*d*) If Y is true, Q is true;

(*e*) If Z is true, R is true;

prove *syllogistically*,

(*f*) If X is false, P is false;

(*g*) If Y is false, Q is false;

(*h*) If Z is false, R is false.

[1] Cf. Mansel's *Aldrich*, p. 103.

CHAPTER VIII.

DISJUNCTIVE SYLLOGISMS.

208. The Disjunctive Syllogism.

A *Disjunctive Syllogism* may be defined as a formal reasoning consisting of two premisses and a conclusion, of which one premiss is disjunctive while the other premiss and the conclusion are categorical[1].

For example,

$$A \text{ is either } B \text{ or } C,$$
$$A \text{ is not } B,$$
therefore, A is C.

The categorical premiss in this example denies one of the alternatives stated in the disjunctive premiss, and we

[1] Archbishop Thomson's definition of the disjunctive syllogism— "An argument in which there is a disjunctive judgment" (*Laws of Thought*, p. 197)—must I think be regarded as too wide. It would include such a syllogism as the following,—

$$B \text{ is either } C \text{ or } D,$$
$$A \text{ is } B,$$
therefore, A is either C or D.

The argument here in no way turns upon the disjunction, and the reasoning may be regarded as an ordinary categorical syllogism in *Barbara*, the major term being complex.

A more general treatment of reasonings involving disjunctive judgments is given in Part IV.

are hence enabled to affirm the other alternative as our con-
clusion. This is called the *modus tollendo ponens*.

Some logicians also recognise as valid a *modus ponendo
tollens*, in which the categorical premiss affirms one of the
alternatives stated in the disjunctive premiss, and the con-
clusion denies the other alternative. Thus,

<div align="center">

A is either B or C,

A is B,

therefore, *A is not C*.

</div>

This proceeds on the assumption that the elements of
the disjunction are mutually exclusive, which in my opinion
is not necessarily the case[1]. The recognition or denial of
the validity of the *modus ponendo tollens* depends then upon
our interpretation of the disjunctive proposition itself.

209. Comment upon the following definitions of
a disjunctive syllogism :—

"A disjunctive syllogism is a syllogism of which
the major premiss is a disjunctive and the minor a
simple proposition, the latter affirming or denying one
of the alternatives stated in the former."

"A disjunctive syllogism is a syllogism whose
major premiss is a disjunctive proposition."

210. Examine the question whether the force of
a Disjunctive Proposition as a premiss in an argument
is equivalent to that of a Hypothetical Proposition.

<div align="right">

[L.]

</div>

At any rate so far as the disjunctive syllogism is con-
cerned this question must be answered in the affirmative.

---

[1] Cf. section 109.

A is either B or C,
A is not B,
therefore, A is C;

may be resolved into,—

If A is not B, A is C,
A is not B,
therefore, A is C;

or, into,—

If A is not C, A is B,
A is not B,
therefore, A is C.

It may be observed that those who deny the character of mediate reasoning to the hypothetical syllogism must also deny it to the disjunctive syllogism, or else they must refuse to recognise the resolution of the disjunctive proposition into one or more hypothetical propositions.

211.   Is it possible to apply distinctions of Figure either to Hypothetical or to Disjunctive Syllogisms ?
[C.]

212.   Comment upon Jevons's statement :—" It will be noticed that the disjunctive syllogism is governed by totally different rules from the ordinary categorical syllogism, since a negative premiss gives an affirmative conclusion in the former, and a negative in the latter."

213.   If all things are either X or Y, and all things are either Y or Z, what inference can you draw ?
[Jevons, *Studies*, p. 303.]

214. The Dilemma.

The proper place of the Dilemma among Conditional Arguments is made puzzling by the fact that conflicting definitions of the Dilemma are given by different logical writers. It will be useful to comment briefly upon some of these definitions.

(1) Mansel (*Aldrich*, p. 108) defines the Dilemma as "a syllogism having a conditional (hypothetical) major premiss *with more than one antecedent,* and a disjunctive minor." Equivalent definitions are given by Whately and Jevons.

Three forms of dilemma are recognised by these writers :—

i. The *Simple Constructive* Dilemma.
   If A is B, C is D; and if E is F, C is D;
   But either A is B or E is F;
   Therefore, C is D.

ii. The *Complex Constructive* Dilemma.
   If A is B, C is D; and if E is F, G is H;
   But either A is B or E is F;
   Therefore, Either C is D or G is H.

iii. The *Destructive* Dilemma, (always *Complex*).
   If A is B, C is D; and if E is F, G is H;
   But either C is not D or G is not H;
   Therefore, Either A is not B or E is not F.

The Destructive Dilemma is said to be always complex; and the simple form corresponding to the third of the above is certainly excluded by the definition given. It would run,—

> If A is B, C is D; and if A is B, E is F;
> But either C is not D or E is not F;
> Therefore, A is not B ;

and here there is *only one antecedent* in the major.

But the question arises whether such exclusion is not arbitrary, and whether this definition ought not therefore to be rejected.

Whately regards the name Dilemma as necessarily implying two *antecedents ;* but does it not rather imply two *alternatives*, each of which is equally distasteful? He goes on to assert that the excluded form is merely a destructive hypothetical syllogism, similar to the following,—

> If A is B, C is D ;
> C is not D ;
> therefore, A is not B.

But the two really differ precisely as the simple constructive dilemma,—

> If A is B, C is D; and if E is F, C is D ;
> But either A is B or E is F;
> therefore, C is D ;—

differs from the constructive hypothetical syllogism,—

> If A is B, C is D ;
> A is B ;
> therefore, C is D.

Besides, it is clear that it is not merely a destructive hypothetical syllogism such as has been already discussed, since the premiss which is combined with the hypothetical premiss is not categorical but disjunctive [1].

---

[1] The argument,—

> If A is B, C is D and E is F;
> But either C is not D or E is not F;
> Therefore, A is not B ;

(2) Professor Fowler (*Deductive Logic*, p. 116) gives the following :—"There remains the case in which one premiss of the complex syllogism is a conjunctive, (*i.e.*, a hypothetical), and the other a disjunctive proposition, it being of course understood that the disjunctive proposition deals only with expressions which have already occurred in the conjunctive proposition. This is called a *Dilemma*."

Under this definition, it is no longer required that there shall be at least two antecedents in the hypothetical premiss ; and hence, four forms are included, namely, the two constructive dilemmas, and a simple as well as a complex destructive.

(3) The following definition is sometimes given :— "The Dilemma (or Trilemma or Polylemma) is a syllogism in which two (or three or more) alternatives are given in one premiss, but in the other it is shewn that in any case the same conclusion follows."

This would include the simple constructive dilemma and the simple destructive dilemma, (as already given); but it would not allow that either of the complex dilemmas is

must be distinguished from the following,—

> If A is B, C is D and E is F;
> But C is not D, and E is not F;
> Therefore, A is not B.

In the latter of these there is no alternative given at all, and the reasoning is equivalent to two simple hypothetical syllogisms, yielding the same conclusion, namely,—

> (1) If A is B, C is D ;
>   But C is not D ;
>   Therefore, A is not B.
> (2) If A is B, E is F;
>   But E is not F;
>   Therefore, A is not B.

properly so-called, since in each case we are left with the same number of alternatives in the conclusion as are contained in the disjunctive premiss.

This definition, however, embraces forms that are excluded by both the preceding definitions. For example,

> If A is, either B or C is ;
> But neither B nor C is ;
> Therefore, A is not[1].

(4) Hamilton (*Logic*, I. p. 350) defines the Dilemma as "a syllogism in which the sumption (major) is at once hypothetical and disjunctive, and the subsumption (minor) sublates the whole disjunction, as a consequent, so that the antecedent is sublated in the conclusion." This involved definition appears to have chiefly in view the form last given, namely,—

> If A is, either B is or C is ;
> Neither B is nor C is ;
> Therefore, A is not ;

but it excludes the following,—

> If A is, C is ; and if B is, C is ;
> But either A is or B is ;
> Therefore, C is.

This however is one of the typical forms of Dilemma according to all the preceding definitions.

(5) Thomson (*Laws of Thought*, p. 203) gives the following,—"A dilemma is a syllogism with a conditional (hypothetical) premiss, in which either the antecedent or the consequent is disjunctive."

This definition is probably wider than Thomson himself intended. It would include such forms as the following :—

---

[1] Cf. Ueberweg, *System of Logic*, Lindsay's translation, p. 457.

If *A* is *B* or *E* is *F*, then *C* is *D* ;
But *C* is not *D* ;
Therefore, *A* is not *B*, and *E* is not *F*.
If *A* is *B*, *C* is *D* or *E* is *F* ;
But *A* is *B* ;
Therefore, *C* is *D* or *E* is *F*.

215. " Dilemmatic arguments are more often fal-
lacious than not." Why is this ?   [c.]

Jevons (*Elements of Logic*, p. 168) remarks that " Dilem-
matic arguments are more often fallacious than not, because
it is seldom possible to find instances where two alternatives
exhaust all the possible cases, unless indeed one of them be
the simple negative of the other." In other words, most
dilemmatic arguments will be found to contain a false
premiss. It is however somewhat misleading to say that a
syllogistic argument is fallacious because it contains a false
premiss. At any rate, notwithstanding this, the argument
itself from the point of view of Formal Logic may be per-
fectly cogent.

216. What can be inferred from the premisses,
Either *A* is *B* or *C* is *D*, Either *C* is not *D* or *E* is not *F* ?
Exhibit the reasoning in the form of a dilemma.

THE QUANTIFICATION OF THE PREDICATE.

217. The eight propositional forms resulting from the explicit Quantification of the Predicate.

The fundamental postulate of Logic, according to Sir W. Hamilton, was "that we be allowed to state explicitly in language all that is implicitly contained in thought"; and since he also maintained that "in thought the predicate is always quantified," he made it follow immediately from his postulate, that "in logic, the quantity of the predicate must be expressed, on demand, in language."

This doctrine of the explicit quantification of the predicate led Hamilton to recognise eight distinct propositional forms instead of the customary four :—

All S is all P,	U.
All S is some P,	A.
Some S is all P,	Y.
Some S is some P,	I.
No S is any P,	E.
No S is some P,	η.
Some S is not any P,	O.
Some S is not some P.	ω.

The symbols here attached are due to Thomson[1], and they are the ones in most common use.

---

[1] Thomson however rejects the forms η and ω.

The symbols used by Hamilton himself were *Afa*, *Afi*, *Ifa*, *Ifi*, *Ana*, *Ani*, *Ina*, *Ini*.　Here *f* indicates an affirmative proposition, *n* indicates a negative; *a* means that the corresponding term is distributed, *i* that it is undistributed.

Spalding's symbols (*Logic*, p. 83) are A^2, A, I^2, I, E, $\frac{1}{2}E$, O, $\frac{1}{2}O$.　Mr Carveth Read (*Theory of Logic*, p. 193) suggests A^2, A, I^2, I, E, E_2, O, O_2.

The equivalence of these various symbols is shewn in the following table :—

	Thomson.	Hamilton.	Spalding.	Carveth Read.
All *S* is all *P*	U	*Afa*	A^2	A^2
All *S* is some *P*	A	*Afi*	A	A
Some *S* is all *P*	Y	*Ifa*	I^2	I^2
Some *S* is some *P*	I	*Ifi*	I	I
No *S* is any *P*	E	*Ana*	E	E
No *S* is some *P*	η	*Ani*	$\frac{1}{2}E$	E_2
Some *S* is not any *P*	O	*Ina*	O	O
Some *S* is not some *P*	ω	*Ini*	$\frac{1}{2}O$	O_2

218. The meaning to be attached to the word *some* in the eight propositional forms recognised by Sir William Hamilton.

Professor Baynes, in his authorised exposition of Sir William Hamilton's new doctrine, would at the outset lead

one to suppose that we have no longer to do with the in-
determinate "some" of the Aristotelian Logic, but that this
word is now to be used in the more definite sense of "*some,
but not all.*"   We have seen that the fundamental postulate
of Logic on which Hamilton bases his doctrine is " that we
be allowed to state explicitly in language, all that is im-
plicitly contained in thought"; and applying this postulate,
Mr Baynes (*New Analytic of Logical Forms*) remarks :—
" Predication is nothing more or less than the expression
of the relation of quantity in which a notion stands to an
individual, or two notions to each other.   If this relation
were indeterminate—if we were uncertain whether it was
of part, or whole, or none—there could be no predication.
Since, therefore, the predicate is always quantified in
thought, the postulate applies; *i.e.*, in logic, the quantity of
the predicate must be expressed, on demand, in language.
For example, if the objects comprised under the subject
be some part, but not the whole, of those comprised under
the predicate, we write All X is some Y, and similarly with
other forms."

But if it is true that we know definitely the relative
extent of subject and predicate, and if " some " is used
strictly in the sense of " some but not all," we should have
but *five* propositional forms instead of eight, namely,—*All S
is all P, All S is some P, Some S is all P, Some S is some P*[1],
No S is any P.

We have already shewn (section 95) that the only
possible relations between two terms in respect to their
extension are given by the five diagrams,—

---

[1] Using *some* in the sense here indicated, *Some S is some P* neces-
sarily implies *Some S is not any P* and *No S is some P.*
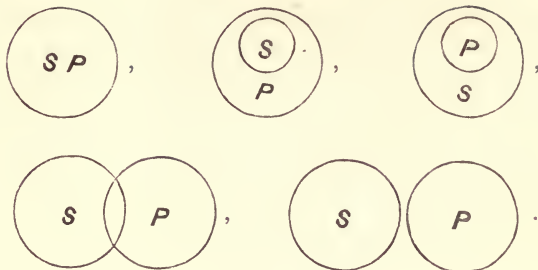
These correspond respectively to the above five propositions; and it is clear that on the view indicated by Mr Baynes the eight forms are redundant. This point is worked out in detail by Mr Venn (*Symbolic Logic*, Chap. I.); he shews the utter inadequacy and unscientific character of the Hamiltonian doctrine.

I am altogether doubtful whether writers who have adopted the eightfold scheme have themselves recognised the pitfalls that surround the use of the word *some*. Many passages might be quoted in which they distinctly adopt the meaning—" some, not all." Thus, Thomson (*Laws of Thought*, p. 150) makes **U** and **A** inconsistent. Bowen (*Logic*, pp. 169, 170) would pass from **I** to **O** by immediate inference [1]. Hamilton himself would agree with Thomson and Bowen on these points; but he is curiously indecisive on the general question here raised. He remarks (*Logic*, II. p. 282) that *some* "is held to be a definite *some* when the other term is definite," *i.e.*, in **A** and **Y**, η and **O** ; but "on the other hand, when both terms are indefinite or particular the *some* of each is left wholly indefinite,"

[1] " This sort of Inference," he says, " Hamilton would call *Integration*, as its effect is, after determining one part, to reconstitute the whole by bringing into view the remaining part."

i.e., in **I** and ω[1]. This is very confusing, and it would be most difficult to apply the distinction consistently. Hamilton himself certainly does not so apply it. For example, on his view it should no longer be the case that two affirmative premisses necessitate an affirmative conclusion; nor that two negative premisses invalidate a syllogism. Thus, the following should be regarded as valid:—

> All *P* is some *M*,
> All *M* is some *S*,
> ———————————
> therefore, Some *S* is not any *P*.

> No *M* is any *P*,
> Some *S* is not any *M*,
> ———————————
> therefore, Some *S* is not any *P*.

Such syllogisms as these, however, are not admitted by Hamilton and Thomson. Hamilton's supreme canon of the categorical syllogism (*Logic*, II. p. 357) is :—" What worse relation of subject and predicate subsists between either of two terms and a common third term, with which one, at least, is positively related ; that relation subsists between the two terms themselves." This clearly provides

---

[1] Mr Lindsay, however, in expounding Hamilton's doctrine (*Appendix to Ueberweg's System of Logic*, p. 580) says more decisively,— " Since the subject must be equal to the predicate, vagueness in the predesignations must be as far as possible removed. *Some* is taken as equivalent to *some but not all*."

Spalding (*Logic*, p. 184) definitely chooses the other alternative. He remarks that in his own treatise "the received interpretation *some at least* is steadily adhered to."

Mr Carveth Read (*Theory of Logic*, p. 196) distinguishes two schemes of what he calls Bidesignate Relationships (Quantified Predicates) in one of which the sign *Some* is understood to mean *Some only*, and in the other *Some at least*. In each case, however, he seems to retain eight distinct propositional forms.

that one premiss at least shall be affirmative, and that an affirmative conclusion should follow from two affirmative premisses. Thomson (*Laws of Thought*, p. 165) explicitly lays down the same rules. Here then is further evidence of the unscientific nature of the Hamiltonian doctrine. The same subject is pursued further in the three following sections.

219. What results would follow if we were to interpret ' Some *A*'s are *B*'s ' as implying that ' Some other *A*'s are not *B*'s ' ?

[Jevons, *Studies in Deductive Logic*, p. 151.]

Professor Jevons himself answers this question by saying, " The proposition ' Some *A*'s are *B*'s ' is in the form **I**, and according to the table of opposition **I** is true if **A** is true ; but **A** is the contradictory of **O**, which would be the form of ' Some other *A*'s are not *B*'s.' Under such circumstances **A** could never be true at all, because its truth would involve the truth of its own contradictory, which is absurd."

This is turning the criticism the wrong way, and proves too much. It is not true that we necessarily involve ourselves in self-contradiction if we use *some* in the sense of *some only*. What should be pointed out is that if we use the word in this sense, the truth of **I** no longer follows from the truth of **A** ; but on the other hand these two propositions are inconsistent with each other.

Taking the five propositional forms which are obtained by attaching this meaning to *some*, namely,—*All S is all P*, *All S is some P*, *Some S is all P*, *Some S is some P*, *No S is P*,—it should be observed that each one of these propositions is inconsistent with each of the others, and also that no one is the contradictory of any one of the

others. If, for example, on this scheme we wish to express the contradictory of **U**, we can only do so by affirming an alternative between **Y**, **A**, **I** and **E**.

Nothing of all this appears to have been noted by the Hamiltonian writers[1], even in the cases in which they explicitly profess to use *some* in the sense of "*some only.*"

How the above five forms may be expressed by means of the ordinary Aristotelian four forms has been discussed in section 99.

220. If in the eight Hamiltonian forms of proposition *some* is used in the ordinary logical sense, what is the precise information given by each of these propositions?

Taking the five possible relations between two terms, and numbering them as follows,—
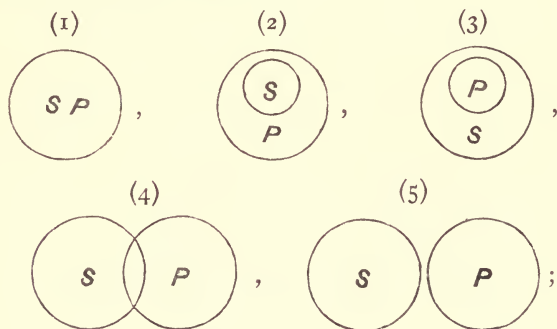
we may write against each of the propositional forms the relations which are compatible with it[2]:—

---

[1] Thomson (*Laws of Thought*, p. 149) gives a scheme of opposition in which **I** and **E** appear as contradictories, but **A** and **O** as contraries. He appears to use *some* in the sense of *some but not all* in the case of **A** and **Y** only.

[2] If the Hamiltonian writers had attempted to illustrate their doc-

U	I
A	I, 2
Y	I, 3
I	I, 2, 3, 4
E	5
η	2, 4, 5
O	3, 4, 5
ω	I, 2, 3, 4, 5

We have then the following pairs of contradictories,—
A, O; Y, η; I, E. The contradictory of **U** is obtained
by affirming an alternative between η and **O**.

We may point out how each of the above would be
expressed without the use of quantified predicates :—

$$\mathbf{U} = SaP,\ PaS;$$
$$\mathbf{A} = SaP;$$
$$\mathbf{Y} = PaS;$$
$$\mathbf{I} = SiP;$$
$$\mathbf{E} = SeP;$$
$$\eta = PoS;$$
$$\mathbf{O} = SoP.$$

trine by means of the Eulerian diagrams, they would I think either
have found it to be unworkable, or they would have worked it out to
a more distinct and consistent issue.

What exact information, if any, is given by ω is discussed in the following section.

221. The Hamiltonian proposition ω, " Some S is not some P."

The proposition ω, "Some S is not some P," is not inconsistent with any of the other propositional forms, not even with **U**, "All S is all P." For example, "all equilateral triangles are all equiangular triangles," yet nevertheless "this equilateral triangle is not that equilateral triangle," which is all that ω asserts. "Some S is some P" is indeed always true except when both the subject and the predicate are the name of an individual and the same individual. De Morgan[1] (*Syllabus*, p. 24) points out that its contradictory is,—"S and P are singular and identical; there is but one S, there is but one P, and S is P." It may be said without hesitation that the proposition ω is of absolutely no logical importance.

222. To what extent do the eight forms resulting from predicating of *all* or *some* trains, that they *do* or *do not*, stop at *all* or *some* stations, coincide in significance with Hamilton's schedule? In particular, do the objections to "Some A is not some B" apply to the proposition "Some trains do not stop at some stations"?   [V.]

223. Examine Thomson's statement that "η has the semblance only, and not the power of a denial. True though it is, it does not prevent our making another judgment of the affirmative kind, from the same terms."

---

[1] De Morgan in several passages criticizes with great acuteness the Hamiltonian scheme of propositions.

224. Write out the various judgments, including **U** and **Y**, which are logically opposed to the judgment : No puns are admissible. State in the case of each judgment thus formed what is the kind of opposition in which it stands to the original judgment, and also the kind of opposition between each pair of the new judgments. [C.]

225. Explain precisely how it is that **O** admits of ordinary conversion if the principle of the Quantification of the Predicate is adopted, although not otherwise.

226. Test the validity of the following syllogisms, and examine whether or not the reasoning contained in those that are valid can be expressed without the use of quantified predicates :—

In Figure 1, **UUU IU**η.
In Figure 2, η**UO**.
In Figure 3, **YAY, Y**η**E**.

(1) **UUU** in Figure 1 is valid :—

All M is all P,
All S is all M,
therefore, *All S is all P.*

It should be noticed that whenever one of the premisses is **U**, the conclusion may be obtained by substituting S or P (as the case may be) for M in the other premiss.

Without the use of quantified predicates, the above reasoning may be expressed by means of the two syllogisms,—

All M is P,	*All M is S,*
All S is M,	*All P is M,*
therefore, *All S is P.*	therefore, *All P is S.*

(2) **IU**η in Figure 1 is invalid, if *some* is used in its ordinary logical sense. The premisses are *Some M is some P*, and *All S is all M*. We may therefore obtain the legitimate conclusion by substituting S for M in the major premiss. This yields *Some S is some P*.

If, however, *some* is here used in the sense of *some only*, *No S is some P* follows from *some S is some P*, and the original syllogism is valid, although a negative conclusion is obtained from two affirmative premisses.

This syllogism is given valid by Thomson (*Laws of Thought*, p. 188); but apparently only through a misprint for **IE**η. Using *some* in the sense of *some only*, several other syllogisms would be valid that he does not give as such[1].

(3) η**UO** in Figure 2 is valid:—

> *No P is some M*,
> *All S is all M*,
> therefore, *Some S is not any P*.

Without the use of quantified predicates, we can obtain an equivalent argument in *Bocardo*, thus,—

> *Some M is not P*,
> *All M is S*,
> therefore, *Some S is not P*.

(4) **YAY** in Figure 3 is valid:—

> *Some M is all P*,
> *All M is some S*,
> therefore, *Some S is all P*.

Without quantified predicates the reasoning may be expressed in *Barbara*, thus,—

---

[1] Cf. section 218.

> *All M is S,*
> *All P is M,*
> therefore, *All P is S.*

(5) **YηE** in Figure 3 is invalid :—

From　　　　*Some M is all P,*
and　　　　*No M is some S,*
we infer that　　　　*No S is any P ;*

but this involves illicit process of the minor.

227.　Examine the validity of the following moods :—

In Figure 1, **UAU, YOO, EYO** ;
In Figure 2, **AAA, AYY, UOω** ;
In Figure 3, **YEE, OYO, AωO.**　　　[C.]

228.　In what figures, if any, are the following moods valid ? Where the conclusion is weakened, point out the fact :—

AUI ; YAY ; UOη ; IUη ; UEO.　　　[L.]

229.　Is it possible that there should be three propositions such that each in turn is deducible from the other two ?　　　[V.]

230.　The Figured and the Unfigured Syllogism.

The distinction between the figured and the unfigured syllogism is due to Hamilton, and is connected with his doctrine of the Quantification of the Predicate.

By a rigid quantification of the predicate the distinction between subject and predicate may be dispensed with ; and such being the case there is no ground left for distinction of figure, (which depends upon the position of the middle term as subject or predicate in the premises). This

gives what Hamilton calls the *Unfigured Syllogism*. For
example,—

    Any bashfulness and any praiseworthy are not equivalent,
    All modesty and some praiseworthy are equivalent,
therefore, Any bashfulness and any modesty are not equi-
    valent.

    All whales and some mammals are equal,
    All whales and some water animals are equal,
therefore, Some mammals and some water animals are
    equal.

    There is an approach here towards the Equational Logic.

    Hamilton gives a distinct canon for the unfigured
syllogism as follows :—" In as far as two notions either
both agree, or one agreeing the other does not, with a
common third notion ; in so far these notions do or do not
agree with each other."

231. Examine technically the following arguments :—

(1) Those who hold that the Insane should not be punished ought in consistency to admit also that they should not be threatened; for it is clearly unjust to punish any one without previously threatening him.

(2) If he pleads that he did not steal the goods, why, I ask, did he hide them, as no thief ever fails to do ?   [v.]

232. Examine technically the following arguments :—

Knavery and folly always go together ; so, knowing him to be a fool I distrusted him.

If I deny that poverty and virtue are inconsistent, and you deny that they are inseparable, we can at least agree that some poor are virtuous.

How can you deny that the infliction of pain is justifiable if punishment is sometimes justifiable and yet always involves pain ?   [v.]

233. Test the following :—

" If all men were capable of perfection, some would have attained it ; but, none having done so, none are capable of it." [v.]

234. Examine the following reasoning :—

How can you deny that any poor should be relieved, when you deny that sickness and poverty are inseparable, and also that any sick should not be relieved ? [v.]

235. In how many different syllogistic moods could you express the reasoning in the following sentence by supplying the proper premises ?

These plants cannot be orchids, for they have opposite leaves. [v.]

236. In how many different moods may the argument implied in the following proposition be stated ?

" No one can maintain that all persecution is justifiable who admits that persecution is sometimes ineffective."

How would the formal correctness of the reasoning be affected by reading " deny" for " maintain" ?

[v.]

237. What conclusions (if any) can be drawn from each pair of the following sentences taken two and two together ?

(1) None but gentlemen are members of the club ;

(2) Some members of the club are not officers;

(3) All members of the club are invited to compete;

(4) All officers are invited to compete.

Point out the mood and figure in each case in which you make a valid syllogism; and state your reasons when you consider that no valid syllogism is possible.   [V.]

238. " No wise man is unhappy; for no dishonest man is wise, and no honest man is unhappy."

Examine this inference, and if you think it sound resolve it into a regular syllogism.   [W.]

239. Detect the fallacy in the following argument :—

" A vacuum is impossible, for if there is nothing between two bodies they must touch."   [N.]

240. Write the following arguments in syllogistic form, and reduce them to Figure 1 :—

(*a*) Falkland was a royalist and a patriot; therefore, some royalists were patriots.

(*β*) All who are punished should be responsible for their actions; therefore, if some lunatics are not responsible for their actions, they should not be punished.

(*γ*) All who have passed the Little-Go have a knowledge of Greek; hence *A. B.* cannot have passed the Little-Go, for he has no knowledge of Greek.

262 SYLLOGISMS. [PART III.

241. Whately says,—"'Every true patriot is disinterested, few men are disinterested, therefore few men are true patriots,' might appear at first sight to be in the second figure and faulty; whereas it is *Barbara* with the premisses transposed."

Do you consider this resolution of the above syllogism to be the correct one?

242. Examine the validity of the following arguments:—

(*a*) Old Parr, healthy as the wild animals, attained the age of 152 years; all men might be as healthy as the wild animals; therefore, all men might attain to the age of 152 years.

(*β*)   Most M is P,
                    Most S is M,
          therefore, Some S is P.

243. Examine the validity of the following arguments:—

(i) Since the end of poetry is pleasure, that cannot be unpoetical with which all are pleased.

(ii) It is quite absurd to say "I would rather not exist than be unhappy," for he who says "I will this, rather than that," chooses something. Non-existence, however, is no something, but nothing, and it is impossible to choose rationally when the object to be chosen is nothing.

244. Can the following arguments be reduced to syllogistic form?

(1)   The sun is a thing insensible;

The Persians worship the sun;

Therefore, the Persians worship a thing insensible.

(2)   The Divine law commands us to honour
kings;

Louis XIV. is a king;

Therefore, the Divine law commands us to honour
Louis XIV.   [*Port Royal Logic.*]

245.   Examine the following arguments; where
they are valid, reduce them if you can to syllogistic
form; and where they are invalid, explain the nature
of the fallacy:—

(1)   We ought to believe the Scripture;

Tradition is not Scripture;

Therefore, we ought not to believe tradition.

(2)   Every good pastor is ready to give his life
for his sheep;

Now, there are few pastors in the present day who
are ready to give their lives for their sheep;

Therefore, there are in the present day few good
pastors.

(3)   Those only who are friends of God are happy;

Now, there are rich men who are not friends of
God;

Therefore, there are rich men who are not happy.

(4)   The duty of a Christian is not to praise those
who commit criminal actions;

Now, those who engage in a duel commit a criminal action ;

Therefore, it is the duty of a Christian not to praise those who engage in duels.

(5)   The gospel promises salvation to Christians ;
Some wicked men are Christians ;

Therefore, the gospel promises salvation to wicked men.

(6)   He who says that you are an animal speaks truly ;

He who says that you are a goose says that you are an animal ;

Therefore, he who says that you are a goose speaks truly.

(7)   You are not what I am ;
I am a man ;
Therefore, you are not a man.

(8)   We can only be happy in this world by abandoning ourselves to our passions, or by combating them ;

If we abandon ourselves to them, this is an unhappy state, since it is disgraceful, and we could never be content with it ;

If we combat them, this is also an unhappy state, since there is nothing more painful than that inward war which we are continually obliged to carry on with ourselves ;

Therefore, we cannot have in this life true happiness.

(9)   Either our soul perishes with the body, and thus, having no feelings, we shall be incapable of any evil; or if the soul survives the body, it will be more happy than it was in the body ;

Therefore, death is not to be feared.

                                        [*Port Royal Logic.*]

246.   Examine the following arguments :—

(1)   " He that is of God heareth my words : ye therefore hear them not, because ye are not of God."

(2)   All the fish that the net inclosed were an indiscriminate mixture of various kinds : those that were set aside and saved as valuable, were fish that the net enclosed : therefore, those that were set aside and saved as valuable, were an indiscriminate mixture of various kinds.

(3)   Testimony is a kind of evidence which is very likely to be false : the evidence on which most men believe that there are pyramids in Egypt is testimony: therefore, the evidence on which most men believe that there are pyramids in Egypt is very likely to be false.

(4)   If Paley's system is to be received, one who has no knowledge of a future state has no means of distinguishing virtue and vice : now one who has no means of distinguishing virtue and vice can commit no sin : therefore, if Paley's system is to be received, one who has no knowledge of a future state can commit no sin.

(5)   If Abraham were justified, it must have been either by faith or by works : now he was not justified

by faith (according to James), nor by works (according to Paul): therefore, Abraham was not justified.

(6) For those who are bent on cultivating their minds by diligent study, the incitement of academical honours is unnecessary ; and it is ineffectual, for the idle, and such as are indifferent to mental improvement : therefore, the incitement of academical honours is either unnecessary or ineffectual.

(7) He who is most hungry eats most ; he who eats least is most hungry : therefore, he who eats least eats most.

(8) A monopoly of the sugar-refining business is beneficial to sugar-refiners : and of the corn-trade to corn-growers : and of the silk-manufacture to silk-weavers, &c., &c.; and thus each class of men are benefited by some restrictions. Now all these classes of men make up the whole community : therefore a system of restrictions is beneficial to the community.

[Whately.]

247. The following are a few examples in which the reader can try his skill in detecting fallacies, determining the peculiar form of syllogisms, and supplying the suppressed premisses of enthymemes. Several of the examples contain more than one syllogism.

(1) None but those who are contented with their lot in life can justly be considered happy. But the truly wise man will always make himself contented with his lot in life, and therefore he may justly be considered happy.

(2)   All intelligible propositions must be either true or false.   The two propositions "Cæsar is living still," and "Cæsar is dead," are both intelligible propositions ; therefore they are both true, or both false.

(3)   Many things are more difficult than to do nothing.   Nothing is more difficult to do than to walk on one's head.   Therefore, many things are more difficult than to walk on one's head.

(4)   None but Whigs vote for Mr B.   All who vote for Mr B. are ten-pound householders.   Therefore none but Whigs are ten-pound householders.

(5)   If the Mosaic account of the cosmogony is strictly correct, the sun was not created till the fourth day.   And if the sun was not created till the fourth day, it could not have been the cause of the alternation of day and night for the first three days. But either the word "day" is used in Scripture in a different sense to that in which it is commonly accepted now, or else the sun must have been the cause of the alternation of day and night for the first three days.   Hence it follows that either the Mosaic account of the cosmogony is not strictly correct, or else the word "day" is used in Scripture in a different sense to that in which it is commonly accepted now.

(6)   Suffering is a title to an excellent inheritance; for God chastens every son whom He receives.

(7)   It will certainly rain, for the sky looks very black.   [Solly, *Syllabus of Logic.*]

248. Examine the following arguments:

(1) All the householders in the kingdom, except women, are legally electors, and all the male householders are precisely those men who pay poor-rates; it follows that all men who pay poor-rates are electors.

(2) All men are mortals, and all mortals are those who are sure to die; therefore, all men are all those who are sure to die.

[Jevons, *Studies*, p. 162.]

249. State the following arguments in Logical form, and examine their validity:—

(1) Poetry must be either true or false: if the latter, it is misleading; if the former, it is disguised history, and savours of imposture as trying to pass itself off for more than it is. Some philosophers have therefore wisely excluded poetry from the ideal commonwealth.

(2) If we never find skins except as the teguments of animals, we may safely conclude that animals cannot exist without skins. If colour cannot exist by itself, it follows that neither can anything that is coloured exist without colour. So if language without thought is unreal, thought without language must also be so.

(3) Had an armistice been beneficial to France and Germany, it would have been agreed upon by those powers; but such has not been the case; it is plain therefore that an armistice would not have been advantageous to either of the belligerents.

(4)   If we are marked to die, we are enow
       To do our country loss: and, if to live,
       The fewer men, the greater share of honour.
                                                    [O.]

250.   Dr Johnson remarked that "a man who
sold a penknife was not necessarily an ironmonger."
Against what logical fallacy was this remark directed?
                                                    [C.]

251.   Exhibit the following in syllogistic form;
naming the mood and figure; when possible, reduce
them to the first figure: (a) The disciples of Wagner
overrate him, for he has caused a great reform in
dramatic art, and all great reformers are over-esti-
mated by their followers.   (b) Some undergraduates
are guilty of conduct to which no gentleman would
stoop; so some undergraduates are not gentlemen.
(c) Not all the things we neglect are worthless, for
some truths are neglected and none without value.
                                                    [C.]

252.   Examine on logical principles the following
arguments; and, if you find any fallacies, name them:

(a)   The existence of State-officials is unjustifi-
able: for since men are by nature equal, it is con-
trary to nature that one should govern another.

(b)   Instinct and reason are opposed: so a good
action, if instinctive, is the opposite of that which
reason would dictate.   [C.]

253.   Put the following propositions into their
simplest Logical form; name the Syllogistic Moods

in which they can be proved; and find premisses that
in some Mood will prove them :

(1)   Not all the unhappy are evildoers.

(2)   Only the wise are free.   [C.]

254. Examine the following arguments, pointing
out any fallacies that they contain :

(*a*)   The more correct the logic, the more certainly
will the conclusion be wrong if the premisses are
false.   Therefore, where the premisses are wholly un-
certain the best logician is the least safe guide.

(*b*)   The spread of education among the lower
orders will make them unfit for their work: for it has
always had that effect on those among them who
happen to have acquired it in previous times.

(*c*)   This pamphlet contains seditious doctrines.
The spread of seditious doctrines may be dangerous
to the State.   Therefore, this pamphlet must be sup-
pressed.   [C.]

255. "To prove that Dissent is wrong you must
appeal to the authority of the Church, and this you
must base on the Bible ; and you must also deny the
supremacy of Conscience.   Moreover you, at least, as
an Anglican, must ignore the Reformation."

How should you draw out fully the argument here
implied?   To what extent does it naturally fall into
syllogistic form ?   [V.]

256. No one can maintain that all republics
secure good government who bears in mind that

good government is inconsistent with a licentious press.

What premises must be supplied to express the above reasoning in *Ferio*, *Festino* and *Ferison* respectively?   [V.]

257. Using any of the forms of Immediate Inference, shew in how many moods the following argument can be expressed :—" Every law is not binding, for some laws are morally bad, and nothing which is so is binding."   [L.]

258. State the following reasonings in strict logical form, and estimate their validity :—

(*a*) As thought is existence, what contains no element of thought must be non-existent.

(*b*) Since the laws allow everything that is innocent, and avarice is allowed, it is innocent.

(*c*) Timon being miserable is an evil-doer, as happiness springs from well-doing.   [L.]

259. Comment carefully upon the following statements :—

" The most perfect Logic will not serve a man who starts from a false premiss."

" I am enough of a logician to know that from false premisses it is impossible to draw a true conclusion."   [L.]

260. Might I be satisfied that a particular war was a just one, assuming (what was the fact) that it was popular, and also (what is more doubtful) that all just wars are popular?

Are honours and rewards, public or private, to be pronounced useless, because they cannot influence the stupid, and men of genius rise above them ?

Because some persons in the dark cannot help thinking of ghosts, though they do not believe in them, does it follow that it is absurd to maintain that, when we cannot avoid thinking or conceiving of a thing, it must be true ?   [L.]

CHAPTER XI.

261. Prove by means of the syllogistic rules that, given the truth of one premiss and of the conclusion of a valid syllogism, the knowledge thus in our possession is in no case sufficient to prove the truth of the other premiss.

We have to shew that if one premiss and the conclusion of a valid syllogism be taken as a new pair of premisses they do not in any case suffice to establish the other premiss.

The premiss given true must be affirmative, for if it is negative, the original conclusion will be negative, and combining these we shall have two negative premisses which can yield no conclusion.

The middle term must be distributed in the premiss given true, for if not it must be distributed in the other premiss, but this being the conclusion of the new syllogism, it must *also* be distributed in the premiss given true or we shall have an illicit process in the new syllogism.

Therefore, the premiss given true, being affirmative, and distributing the middle term, cannot distribute the other term which it contains. Neither therefore can this term be distributed in the original conclusion. But this is the

term which will be the middle term of the new syllogism, and *we shall therefore have undistributed middle.*

The given syllogism then being valid, we have shewn it to be impossible that a new syllogism having one of the original premisses and the original conclusion for its premisses, with the other original premiss for its conclusion, can be valid also[1].

262. Given that in a valid syllogism one premiss is false and the other true, shew that *in no case* will this suffice to prove the conclusion false[2].

This might be established by taking all possible syllogisms, and shewing that the statement holds true with regard to each in turn; but this method is clearly to be avoided if possible.

It might also be deduced from the proposition established in the preceding example. Let the premisses of a valid syllogism be P and Q and the conclusion R. P and the contradictory of Q will not prove the contradictory of R; for if so it would follow that P and R would prove Q; but this has been shewn not to be the case.

Another easy solution is obtainable by assuming that

---

[1] Other methods of solution more or less distinct from the above might be given. A somewhat similar problem is discussed by Solly, *Syllabus of Logic*, pp. 123—126, 132—136. Hamilton (*Logic*, I. p. 450) considers the doctrine "that if the conclusion of a syllogism be true, the premisses may be either true or false, but that if the conclusion be false, one or both of the premisses must be false" to be extra-logical, if it is not absolutely erroneous. He is clearly wrong, since the doctrine in question admits of a purely formal proof.

[2] This problem might also be stated as follows,—Shew that if for one of the premisses of a valid syllogism we substitute its contradictory, this will not in any case enable us to establish the contradictory of the original conclusion.

the given syllogism is reduced to Figure 1. After such re-
duction, it will, in accordance with the special rules of
Figure 1, have a universal major and an affirmative minor.
Then since the contradictory of a universal is particular and
of an affirmative negative, if either premiss is given false
we have in its place either a particular major or a negative
minor. But, (since the syllogism is still in Figure 1), in
neither of these cases can we draw any conclusion at all,
and therefore *a fortiori* we cannot infer that the original
conclusion is false.

I add an *outline* of an independent general solution of
the given problem[1].

Let the following symbols be used:—

T = premiss given true;

F = premiss given false;

C = original conclusion;

F' = contradictory of F;

C' = contradictory of C;

a = original syllogism;

β = syllogism of which the premisses are T and F', and
the conclusion C';

P = major term;
M = middle term; } These will be the same both in
S = minor term. } a and β.

We have to shew that β cannot be a valid syllogism.

T cannot be particular, for in this case F' would also
be particular.

T cannot be negative, for in this case F' would also be
negative.

T then must be universal affirmative.

----

[1] Several steps are omitted, but these the student should carefully
fill in for himself.

(1) Let F also be universal affirmative.

We may shew that C must also be universal, (*i.e.*, a cannot have a weakened conclusion); and it must of course be affirmative.

Then in a, S and M must be distributed;
        in β, P and M must be distributed.

But if F distributed M, M cannot be distributed in β; and if F distributed S, P cannot be distributed in β.

(2) Let F be universal negative. We may again shew that C must be universal.

In this case T cannot distribute M; but neither can F' distribute M.

(3) Let F be particular affirmative.

C' will be universal negative. Therefore, in β we must distribute S, M, P.

But T must distribute M; it cannot therefore distribute S or P, one of which must therefore be undistributed in β.

(4) Let F be particular negative.

In a, M and P must be distributed;
in β, M and S must be distributed.

But if F distributed M, M cannot be distributed in β; and if F distributed P, S cannot be distributed in β.

263. Given a valid syllogism in Figure 1, is there any case in which the mere knowledge that we may start from the contradiction of its premisses will furnish premisses for another valid syllogism?

264. An apparent syllogism of the second figure with a particular premiss is found to break the general rules of the syllogism in this particular only, that the middle term is undistributed. If the particular pre-

miss is false and the other true, what do we know about the truth or falsity of the conclusion ?

Can an apparent syllogism break all the rules of syllogism at once ?

265. Given the two following statements *false :—* (i) either all M is all P, or some M is not P; (ii) some S is not M;—what is all that you can infer, (*a*) with regard to S in terms of P; (*b*) with regard to P in terms of S?

266. If (1) it is false that whenever X is found Y is found with it, and (2) not less untrue that X is sometimes found without the accompaniment of Z, are you justified in denying that (3) whenever Z is found there also you may be sure of finding Y? And however this may be, can you in the same circumstances judge anything about Y in terms of Z? [R.]

267. If whenever X is present, Z is not absent, and sometimes when Y is absent, X is present, but if it cannot be said that the absence of X determines anything about either Y or Z, can anything be determined as between Z and Y? [R.]

268. If B is always found to coexist with A, except when X is Y, (which it commonly, though not always, is), and if, even in the few cases where X is not Y, C is never found absent without B being absent also, can you make any other assertion about C? [R.]

269. From P follows Q; and from R follows S; but Q and S cannot both be true; shew that P and R cannot both be true.   (De Morgan.)

270. Given a syllogism, shew in what cases it is possible to reach the same conclusion by substituting for the middle term its contradictory.   [W.]

[We are supposed here to perform immediate inferences upon our premisses so as to obtain a new middle term which is the contradictory of the original middle term.]

271. What conclusion can be drawn from the following propositions?

The members of the board were all either bond-holders or shareholders, but not both; and the bond-holders, as it happened, were all on the board. [V.]

We have given,—

No member of the board is both a bondholder and a shareholder,

All bondholders are members of the board;
and these premisses yield a conclusion (in *Celarent*),

No bondholder is both a bondholder and a shareholder,
that is, No bondholder is a shareholder.

272. The following rules were drawn up for a club :—

(i) The financial committee shall be chosen from amongst the general committee;

(ii) No one shall be a member both of the general and library committees, unless he be also on the financial committee;

(iii) No member of the library committee shall be on the financial committee.

Is there anything self-contradictory or superfluous in these rules?

[VENN, *Symbolic Logic*, pp. 261—264.]

Let F = member of the financial committee,

G = member of the general committee,

L = member of the library committee.

The above rules then become,—

(i)   All F is G ;

(ii) If L is G, it is F ;

(iii) No L is F.

From (ii) and (iii) we obtain

(iv) No L is G.

The rules may therefore be written,

(1) All F is G,

(2) No L is G,

(3) No L is F.

But in this form (3) is deducible from (1) and (2).

All that is contained therefore in the rules as originally stated may be expressed by (1) and (2); that is, the rules as originally stated were partly superfluous, and they may be reduced to

(1) The financial committee shall be chosen from amongst the general committee ;

(2) No one shall be a member both of the general and library committees.

If (ii) is interpreted as implying that there *are* individuals who are on both the general and library committees, then it follows that (ii) and (iii) are inconsistent with each other.

273. Are assumptions with regard to "existence" involved in any of the syllogistic processes?

We may as in section 104 take three distinct suppositions with regard to the existential implication of propositions, and proceed to answer the above question on the basis of each in turn. The three suppositions are:—

(1) All propositions imply the existence both of their subjects, and of their predicates.

(2) No propositions imply the existence either of their subjects or of their predicates.

(3) Particular propositions imply the existence of their subjects; but universal propositions do not.

First, we may take the supposition that *every proposition implies the existence both of its subject and of its predicate.* In this case, the existence of the major, middle and minor terms is guaranteed by the premisses, and therefore no further assumption with regard to existence is required in order that the conclusion may be legitimately obtained[1].

Secondly, we may take the supposition that *no proposition logically implies the existence either of its subject or of its predicate.* Let the major, middle and minor terms be respectively P, M, S. The conclusion will imply that if there is any S there is some P or not-P, (according as it is affirmative or negative). Will the premisses also necessarily imply this?

It has been shewn in section 141 that a universal affirmative conclusion, All S is P, can only be proved by means of the premisses,—All M is P, All S is M; and it is clear that these premisses themselves necessarily imply that

---

[1] If however we are to be allowed to proceed as in section 123, (where from all P is M, all S is M, we inferred that some not-S is not-P), we must posit the existence not merely of the terms directly involved, but also of their contradictories.

if there is any S there is some P. No assumption then with regard to existence is involved in syllogistic reasoning if the conclusion is universal affirmative.

Again, as shewn in section 141, a universal negative conclusion, No S is P, can only be proved in the following ways,—

> (i)   No M is P, (or No P is M),
>   All S is M,
> therefore, No S is P.
>
> (ii)   All P is M,
>   No S is M, (or No M is S),
> therefore, No S is P.

In (i) the minor premiss implies that if S exists then M exists, and the major premiss that if M exists then not-P exists.

In (ii) the minor premiss implies that if S exists then not-M exists, and the major premiss that if not-M exists then not-P exists, (as shewn in section 104).

It follows then that no assumption is involved if the conclusion is universal negative.

Next, let the conclusion be particular. The implication of the conclusion with regard to existence is now contained in the premisses themselves, if the minor premiss is affirmative, and if the minor term is the subject of the minor premiss, and the middle term the subject of the major premiss, (*i.e.*, if the syllogism is in Figure 1). The same will be found to hold good on special examination of the moods of Figure 2 which yield particular conclusions. But it is otherwise with regard to the moods of Figures 3 and 4. Take, for example, a syllogism in *Darapti*,—

> All M is P,
> All M is S,
> therefore, Some S is P.

The conclusion implies that if S exists P exists; but consistently with the premises, S may be existent while M and P are both non-existent. An implication is therefore contained in the conclusion which is not contained in the premises themselves.

Our results may now be summed up as follows:—On the supposition that no proposition logically implies the existence either of its subject or of its predicate, *we do not require to make any assumption with regard to existence in any syllogistic process yielding a universal conclusion in whatever figure it may be, nor in any syllogistic process yielding a particular conclusion provided it is in Figure* 1 *or Figure* 2 ; but it is otherwise if a particular conclusion is obtained in Figure 3 or Figure 4.

Thirdly, taking the supposition that particular propositions imply the existence of their subjects, although universal propositions do not, it will be found that assumptions with regard to existence are involved in syllogistic reasoning in the following and only in the following cases,—

(i)   In Figures 2 and 4, if the conclusion is particular ;

(ii)   In Figures 1 and 3, if the minor premiss is universal and the conclusion particular.

The student should for himself fill in the steps necessary to establish this conclusion.

274.   " Whatever P and Q may stand for, we may shew *a priori* that some P is Q.   For All PQ is Q by the law of identity, and similarly All PQ is P; therefore, by a syllogism in *Darapti,* some P is Q." How would you deal with this paradox ?

A solution is afforded by the discussion contained in the preceding section ; and this example seems to shew that the enquiry,—how far assumptions with regard to exist-

ence are involved in syllogistic processes,—is not irrelevant
or unnecessary.

275. If *P* is *Q*, and *Q* is *R*, it follows that *P* is
R; but suppose it to be discovered that no such thing
as *Q* exists,—How is the truth of the conclusion, *P*
is *R*, affected by this discovery?　　　　　　[L.]

276. De Morgan says:—" In all syllogisms the
existence of the middle term is a *datum*." Inquire
into the accuracy of this assertion. What does
existence here mean?　　　　　　[L.]

277. On the supposition that no proposition
logically implies the existence either of its subject or
of its predicate, find in what cases of the *Reduction*
of Syllogisms to Figure 1 assumptions with regard
to existence are involved.

278. Given that the middle term is distributed
twice in the premisses of a syllogism, determine
directly, (*i.e.*, without any reference to the special
rules of the figures, or the possible moods in each
figure), in what different moods it might possibly be.

The premisses must be either both affirmative, or one
affirmative and one negative.

In the first case, both premisses being affirmative can dis-
tribute their subjects only. The middle term must therefore
be the subject in each, and both must be universal. This
limits us to the one syllogism,—

<div align="center">

All *M* is *P*,

All *M* is *S*,

therefore, Some *S* is *P*.

</div>

In the second case, one premiss being negative, the conclusion must be negative and will therefore distribute the major term. Hence, the major premiss must distribute the major term, and also (by hypothesis) the middle term. This condition can be fulfilled only by its being one or other of the following,—No *M* is *P*, or No *P* is *M*. The major being negative, the minor must be affirmative, and in order to distribute the middle term it must be All *M* is *S*.

In this case then we get two syllogisms, namely,—

<div align="center">

No *M* is *P*,

All *M* is *S*,

---

therefore, Some *S* is not *P*.

No *P* is *M*,

All *M* is *S*,

---

therefore, Some *S* is not *P*.

</div>

The given condition limits us therefore to three syllogisms, (one affirmative and two negative); and by reference to the mnemonic verses we may now identify these with *Darapti* and *Felapton* in Figure 3, and *Fesapo* in Figure 4.

279. If the major premiss is affirmative, and if the major term is distributed both in premisses and conclusion, while the minor term is undistributed in both, determine *directly* the mood and figure.   [N.]

280. If the major term be distributed in the premisses and undistributed in the conclusion, determine *directly* the mood and figure.   [C.]

[Professor Jevons gives this question in the form: "If the major term be universal in the premisses and particular in the conclusion, determine the mood and figure, it being understood that the conclusion is not a weakened one"

(*Studies in Deductive Logic*, p. 103) ; but the condition here introduced seems unnecessary, since we are in any case limited to a single syllogism.]

281. Given a valid syllogism with two universal premisses and a particular conclusion, such that if its subaltern is substituted for either of the premisses the same conclusion cannot be inferred, determine the mood and figure of the syllogism.

If there is such syllogism, let S, M, P be its minor, middle and major terms respectively.

Since the conclusion is given particular it must be either Some S is P, or Some S is not P.

First, if possible, let it be Some S is P.

The only term which we require to distribute in the premisses is M. But since we have two universal premisses, *two* terms must be distributed in them as subjects[1]. One of these must be superfluous ; and therefore for one of the premisses we may substitute its subaltern, and still get the same conclusion.

The conclusion cannot then be Some S is P.

Secondly, if possible, let the conclusion be Some S is not P.

If the subject of the minor premiss is S, we may clearly substitute its subaltern without affecting the conclusion. The subject of the minor premiss must therefore be M, which will thus be distributed in this premiss. M cannot also be distributed in the major, or else it is clear that its subaltern might be substituted for the minor and neverthe-

---

[1] We here include the case in which the middle term is itself twice distributed.

less the same conclusion inferred. The major premiss must therefore be affirmative with M for its predicate. This limits us to the syllogism,—

<div style="text-align:center">

All P is M,

No M is S,

therefore, Some S is not P;

</div>

and this syllogism, which is **AEO** in Figure 4, does fulfil the given conditions, for if either premiss is made particular, it becomes invalid.

The above amounts to a general proof of the proposition laid down in section 147. *Every syllogism in which there are two universal premisses with a particular conclusion is a strengthened syllogism, with the one exception of* **AEO** *in Figure* 4.

[In his studies in *Deductive Logic*, p. 105, Jevons gives the following: "Prove that wherever there is a particular conclusion without a particular premiss, something superfluous is invariably assumed in the premisses." The case of **AEO** in Figure 4, however, shews that this needs qualification.]

282. Given two valid syllogisms in the same figure in which the major, middle and minor terms are respectively the same. shew, without reference to the mnemonic verses, that if the minor premisses are subcontraries, the conclusions will be identical.

The minor premiss of one of the syllogisms must be **O,** and the major premiss of this syllogism must therefore be **A** and the conclusion **O.** The middle and the major terms having then to be distributed in the premisses, this syllogism is determined, namely,—

All *P* is *M*,
Some *S* is not *M*,

therefore, Some *S* is not *P*.

Since the other syllogism is to be in the same figure, its minor premiss must be Some *S* is *M* the major must therefore be universal, and in order to distribute the middle term it must be negative. The syllogism then is also determined, namely,—

No *P* is *M*,
Some *S* is *M*,

therefore, Some *S* is not *P*.

The conclusions of the two syllogisms are thus shewn to be identical.

283. Given two valid syllogisms in the same figure in which the major, middle and minor terms are respectively the same, shew, without reference to the mnemonic verses, that if the minor premisses are contradictories, the conclusions will not be contradictories.

284. Is it possible that there should be a valid syllogism such that, each of the premisses being converted, a new syllogism is obtainable giving a conclusion in which the old major and minor terms have changed places ?

Prove the correctness of your answer by general reasoning, and if it is in the affirmative, determine the syllogism or syllogisms fulfilling the given conditions.

If such a syllogism is possible, it cannot have two affirmative premisses, or (since **A** can only be converted *per*

accidens) we should have two particular premisses in the new syllogism.

Therefore, *the original syllogism must have one negative premiss.* This cannot be **O**, since **O** is inconvertible.

Therefore, *one premiss of the original syllogism must be* **E**.

First, let this be the major premiss. Then the minor premiss must be affirmative, and its converse being a particular affirmative will not distribute either of its terms. But this converse will be the *major* premiss of the new syllogism, which also must have a negative conclusion. We should then have illicit major in the new syllogism, and this supposition will not give us the desired result.

Secondly, let the minor premiss of the original syllogism be **E**. The major premiss in order to distribute the old major term must be **A**, with the major term as subject. We get then the following, satisfying the given conditions:—

<div align="center">

All *P* is *M*,

No *M* is *S*, or No *S* is *M*,
</div>

therefore,   No *S* is *P*, or Some *S* is not *P*.

that is, we really have four syllogisms, such that both premisses being converted, thus,—

<div align="center">

No *S* is *M*, or No *M* is *S*,

Some *M* is *P*,—
</div>

we have a new syllogism giving a conclusion in which the old major and minor terms have changed places, namely,

<div align="center">

Some *P* is not *S*.
</div>

Symbolically,—

$$\begin{array}{ccc} PaM, & & SeM, \\ \left.\begin{array}{c} MeS, \\ \text{or } SeM, \end{array}\right\} & \quad & \left.\begin{array}{c} \text{or } MeS, \\ MiP, \end{array}\right. \\ \hline \left.\begin{array}{c} \therefore SeP \\ \text{or } SoP \end{array}\right\} & & \therefore PoS. \end{array}$$

If it had been required to retain the *quantity* of the original conclusion, this must be *SoP*, so that we should have only two syllogisms fulfilling the given conditions.

285. Is it possible that there should be two syllogisms having a common premiss such that their conclusions, being combined as premisses in a new syllogism, may give a universal conclusion? If so, determine what the two syllogisms must be.   [N.]

PART IV.

A GENERALISATION OF LOGICAL PROCESSES IN THEIR APPLICATION TO COMPLEX PROPOSITIONS.

CHAPTER I.

THE COMBINATION OF SIMPLE TERMS.

286. Complex Terms.

A *simple term* may for our present purpose be defined as one which is represented by a single symbol; *e.g.*, A, P, X. The combination of simple terms yields a *complex term*.

Simple terms may be combined (1) conjunctively, or (2) disjunctively.

(1) "What is both A and B" is a complex term resulting from the *conjunctive* combination of the simple terms A and B[1]. It is convenient to denote a complex term of this kind by a simple juxtaposition of the terms involved, thus—

---

[1] This species of complex term is called by Jevons a *combined term* (*Pure Logic*, p. 15). So far as it requires a distinctive name I think I should prefer to call it a *conjunctive term*.

AB. Accordingly the proposition "AB is CD" would be read "Anything that is both A and B is both C and D."

(2) "What is either A or B" is a complex term resulting from the disjunctive combination of the simple terms A and B[1].

In what follows it must be remembered that I have adopted the view, that logically the alternatives in a disjunction, (unless they are formal contradictories), are non-exclusive. Thus, if we speak of anything as being "A or B" we do not exclude the possibility of its being both A and B, (compare section 109). In other words "A or B" does not exclude "AB."

The force of a disjunctive term when it is the subject of a proposition should be carefully noted. "Anything that is either P or Q is R," or "whatever is either P or Q is R," may sometimes for the sake of brevity be written "P or Q is R." The latter expression, however, might also be interpreted to mean "one of the two P or Q is R, but we do not know which"; and in consequence of this possible ambiguity, the more definite mode of statement, "Whatever is either P or Q is R[2]," is to be preferred.

A complex term may of course involve both conjunctive and disjunctive combination: e.g., "AB or CD." It is to be noted that the statement that anything is "A or B and

---

[1] This kind of complex term is called by Jevons a *plural term* (*Pure Logic*, p. 25). So far as it requires a distinctive name I think I should prefer to call it a *disjunctive term*.

[2] The subject of this proposition is to be regarded as a single disjunctive term. The same meaning might be given by saying "P and Q are R," but in this case I should consider that we have two distinct subjects, and two propositions elliptically expressed.

at the same time C or D" is equivalent to the statement
that it is "AC or AD or BC or BD."

We speak of a *proposition* as being complex if either its
subject or its predicate is a complex term.

287.   In a complex term the order of combination
is indifferent.

This is true whether the combination be conjunctive or
disjunctive.

Thus, AB and BA are precisely the same terms.   It is
obviously the same thing if we speak of anything as being
both A and B, or if we speak of it as being both B and A.

Again "A or B" and "B or A" have precisely the same
signification.   It is the same thing to speak of anything as
being A or B as to speak of it as being B or A.

288.   The Opposition of Complex Terms.

We shall find it convenient to denote the contradictory
of any simple term by the corresponding small letter.   Thus
for not-A we write a, for not-B we write b. A and a there-
fore denote between them the whole universe of discourse
(whatever that may be), but they denote nothing in common.
In other words, whatever A may designate, it is necessarily
true that Everything (in the universe of discourse) is A or a;
and that A is not a.   It also follows that Aa necessarily
represents a non-existent class; what is both A and not-A
cannot have a place in any universe.

However complex a term may be, we can always find its
contradictory by applying the criterion laid down in section
28.   "A pair of contradictory terms are so related that
between them they exhaust the entire universe to which
reference is made, whilst there is no individual of which they
can both be at the same time affirmed."

Now whatever is not AB must be either a or b, whilst nothing that *is* AB can be either of these; and *vice versa*.

$$\begin{cases} AB, \\ a \text{ or } b, \end{cases}$$

are therefore a pair of contradictories.

Similarly,

$$\begin{cases} A \text{ or } B, \\ ab, \end{cases}$$

are a pair of contradictories.

If, then, two simple terms are conjunctively combined into a complex term, the contradictory of this complex term is given by disjunctively combining the contradictories of the simple terms. And, conversely, if two simple terms are disjunctively combined into a complex term, the contradictory of this complex term is given by conjunctively combining the contradictories of the simple terms.

In each case, we substitute for the simple terms involved their contradictories, and (as the case may be) change *and* for *or*, or *or* for *and*.

But however complex a term may be, it must consist of a series of conjunctive and disjunctive combinations, and it may be successively resolved into the combination of pairs of relatively simple terms till it is at last shewn to result from the combination of absolutely simple terms. For example,—

$$ABC \text{ or } DE \text{ or } FG$$

results from the disjunctive combination of the pair,—

$$\begin{cases} ABC \text{ or } DE, \\ \quad FG \, ; \end{cases}$$

ABC or DE results from the disjunctive combination of the pair,—

$$\begin{cases} ABC, \\ DE \, ; \end{cases}$$

FG results from the conjunctive combination of the pair,—

$$\begin{cases} F, \\ G ; \end{cases}$$

and similarly we may resolve *ABC, DE*.

We may hence deduce the following general rule for obtaining the contradictory of any complex term :—*For each simple term involved, substitute its contradictory ; everywhere change* and *for* or, *and or for* and [1]. This rule is of simple application, and in what follows will be found to be of very considerable importance. Its full force will be made more apparent later on.

Thus the contradictory of

$$A \text{ or } BC$$
$$\text{is} \quad a \text{ and } (b \text{ or } c),$$
$$\textit{i.e.,} \quad ab \text{ or } ac.$$

The contradictory of

$$ABC \text{ or } ABD$$
$$\text{is} \quad (a \text{ or } b \text{ or } c) \text{ and } (a \text{ or } b \text{ or } d),$$

which, as we shall presently shew [2], is resolvable into

$$a \text{ or } b \text{ or } cd.$$

In such statements as the above, the use of brackets is necessary to avoid ambiguity. Thus, *a* or *b* or *c* and *a* or *b* or *d* might be read *a* or *b* or *ca* or *b* or *d* ; but we really mean that each term in the second set of alternatives is to be conjunctively combined with each term in the first set of alternatives.

---

[1] In applying this rule, the information given by two such propositions as "*X* is *P*," "*Y* is *P*," if stated in the form of a single proposition, must be expressed " What is either *X* or *Y* is *P*," *not* "*X* and *Y* are *P*." Compare section 286.

[2] Cf. section 296.

Two terms may be *inconsistent* without being contradictories; *i.e.*, they cannot both be affirmed of anything, but it may be that there are some things of which neither can be affirmed. Thus, we can say that, whatever A, B and C may stand for, "AB is not bC," (since if AB were bC it would involve something being at the same time both B and not-B); but we cannot say that, whatever A, B and C may stand for, "Everything is AB or bC," (since something might be Abc, which is neither AB nor bC). If a conjunctive term contains a term which is the contradictory of a term contained in another conjunctive term then it follows that these two conjunctive terms are inconsistent.

If two conjunctive terms are such that every term in one has corresponding to it in the other its contradictory, these two terms may be regarded as logical contraries, (compare the definition of contrary terms given in section 28). Thus, AbC, aBc may be spoken of as contraries.

289. The Development of Terms by means of the Law of Excluded Middle.

By the Law of Excluded Middle,

Everthing is B or b,

and therefore, A is AB or Ab.

Again, Everything is C or c;

therefore, AB is ABC or ABc,

and Ab is AbC or Abc;

therefore, A is ABC or ABc or AbC or Abc.

This is called the development of a term with reference to other terms; thus, A is here developed with reference to B and C. Compare Jevons, *Pure Logic*, p. 37. He calls any two alternatives which are the same, except as regards one term in each which are contradictories, a *dual term*. Thus, "AB or Ab" is a dual term as regards B.

THE SIMPLIFICATION OF COMPLEX PROPOSITIONS.

290. Types of Complex Propositions.

Complex Propositions may be divided,—

First, (as in the case of simple propositions), according as they are affirmative or negative;

<div align="center">

e.g.,   All *AB* is *C* or *D*;

No *AB* is *C* or *D*.

</div>

Secondly, (also as in the case of simple propositions), according as they are universal or particular;

<div align="center">

e.g., All *AB* is *C* or *D*;

Some *AB* is *C* or *D*.

</div>

We shall deal very little with particular complex propositions, and it will frequently be found convenient to write universal complex propositions in the indefinite form. Thus, by *AB* is *C* or *D* we understand,—

<div align="center">

All AB is *C* or *D*.

</div>

Thirdly, according as only the subject or only the predicate or both subject and predicate are complex terms;

<div align="center">

e.g.,   *AB* is *C*,

A is *B* or *C*,

AB is *C* or *D*.

</div>

Fourthly, according as there is or is not

(α)   conjunctive combination in the subject,—

> *e.g.*, A is C or D,
> AB is C or D ;

(β)   conjunctive combination in the predicate,—

> *e.g.*, AB is C,
> AB is CD ;

(γ)   disjunctive combination in the subject,—

> *e.g.*, A is CD,
>   Whatever is either A or B is CD ;

(δ)   disjunctive combination in the predicate,—

> *e.g.*, AB is C,
> AB is C or D.

291. The Resolution of Complex into relatively Simple Propositions.

Affirmative. Affirmative complex propositions may be immediately resolved into relatively simple ones, so far as there is conjunctive combination in the predicate, or disjunctive combination in the subject.   Thus,—

(1) X is AB

is obviously resolvable into the two propositions,—

> X is A,
> X is B.

(2)   Whatever is either X or Y is A,

is obviously resolvable into the two propositions,—

> X is A,
> Y is A.

Negative. Negative complex propositions may be immediately resolved into relatively simple ones, so far as there is disjunctive combination either in the subject or in the predicate. Thus,

   (1) Nothing that is either X or Y is A

is obviously resolvable into,—

     No X is A,
     No Y is A.

   (2)   No X is A or B

is obviously resolvable into,—

     No X is A,
     No X is B.

The difference between affirmative and negative propositions here must be carefully noticed. So far as there is conjunctive combination in the subject or disjunctive combination in the predicate of an affirmative proposition, or conjunctive combination either in the subject or in the predicate of a negative proposition, we cannot immediately resolve it into simpler propositions.

Even in these cases, however, complex propositions may be resolved into relatively simple ones in a more roundabout way, namely, by the aid of obversion or contraposition, as will be shewn subsequently. Compare especially chapter v.

292. The Equivalence of Propositions.

Two propositions are equivalent if each can be inferred from the other. Similarly, two sets of propositions are equivalent if every member of each set can be inferred from the other set.

When we omit terms from a proposition, or introduce fresh terms, or when in any way we obtain a proposition or set of propositions from another proposition or set of propositions, we should carefully distinguish two cases:—

First, where the force of the original statement is unaffected, so that we can pass back from the new proposition or propositions to the original proposition or propositions.

Secondly, where the force of the original statement is weakened, so that we cannot pass back from the new proposition or propositions to the original proposition or propositions.

In many cases it is of very great importance to know whether in a process of manipulation we have or have not lost any of the information originally given us.

293. The Omission of Terms from a Complex Proposition, the force of the assertion remaining unaffected.

(1) *It is superfluous for any simple term to appear more than once in a conjunctive term.*

Thus AA merely denotes the class A, ABB merely denotes the class AB. Such terms in their original form are tautologous, and the repetition of the term should therefore be struck out. Compare Boole, *Laws of Thought,* p. 31, and Jevons, *Pure Logic,* p. 15.

(2) *In a series of alternatives it is superfluous for any given alternative to be repeated.*

To say that anything is "A or A" is to say that it is A; to say that anything is "A or BC or BC" is to say that it is "A or BC". The repetition of an alternative should

therefore always be struck out. Compare Jevons, *Pure Logic*, p. 26.

(3) *In a universal negative proposition it is superfluous for the same term to appear in every alternative in the subject and also in every alternative in the predicate, that is, in such a case it may be omitted either from the subject or from the predicate.*

For example, to say that No AB is AC is precisely the same as to say that No AB is C, or that No B is AC. For to say that No AB is AC is the same thing as to deny that anything is $ABAC$; but, as shewn above, the repetition of the term A is superfluous, and the statement may therefore be reduced to the denial that anything is ABC. And this may equally well be expressed by saying No AB is C, or No B is AC. Compare also Chapter III, On the Conversion of Complex Propositions.

Similarly, No AB is AC or AD may be reduced to No AB is C or D, or to No B is AC or AD.

(4) *If in an affirmative proposition a term that appears in every alternative in the subject appears also in any alternative in the predicate, it may be dropped from the latter without affecting the force of the statement.*

$$A \text{ is } AB$$

may, (as shewn in section 291), be resolved into

$$A \text{ is } A,$$
$$A \text{ is } B.$$

But A is A is a merely identical proposition and gives no information.

$$A \text{ is } AB$$

may therefore be reduced to the single proposition

$$A \text{ is } B.$$

Similarly,

$$AB \text{ is } AC \text{ or } BC$$

may be reduced to

$$AB \text{ is } C \text{ or } C,$$

and therefore, as shewn above, to

$$AB \text{ is } C.$$

(5) *If one of a series of alternatives is merely a subdivision of another of the alternatives it may be omitted without destroying any of the force of the original assertion.* In other words, in a disjunctive term, "any alternative may be removed, of which a part forms another alternative," (compare Jevons, *Pure Logic*, p. 26).

Thus, AB is a subdivision of A, and "A or AB" may therefore be reduced to "A."

This may be shewn as follows :—

$$X \text{ is } A \text{ or } AB;$$
$$\text{but, } AB \text{ is } A,$$
$$\text{therefore, } X \text{ is } A \text{ or } A,$$
$$\text{therefore, } X \text{ is } A;$$

and conversely, if X is A, since, by the law of excluded middle, A is AB or Ab, it follows that X is Ab or AB;

$$\text{but, } Ab \text{ is } A,$$
$$\text{therefore, } X \text{ is } A \text{ or } AB.$$

Similarly,

$$X \text{ is } AB \text{ or } ABC \text{ or } BD$$

may be simplified by the omission of ABC, becoming

$$X \text{ is } AB \text{ or } BD.$$

(6) *Any term which represents a non-existent class may obviously be dropped from a series of alternatives without altering the force of the proposition ;* this is the case with

a term which involves a self-contradiction. "Aa" means that which is both A and not-A, but by the law of Contradiction, no such class is possible. Such a term as Aa may therefore always be dropped. It follows, therefore, that if X is A or Bb, X is A; and it is clear that there is here no weakening of the force of the original proposition.

(7) *Reduction of dual terms.* (Compare section 289.)

Another simplification is possible where we have two alternatives, one of which contains a term which is the contradictory of a term contained in the other, the remaining terms in each being the same; *e.g.*, ABC or ABc. These may be replaced by a single term containing only the elements which are common to both the original alternatives, without any of the force of the original proposition being lost. Thus, "ABC or ABc" may be replaced by "AB." For ABC and ABc are both AB; and conversely, by the law of excluded middle, AB is ABC or ABc.

Thus, X is AB or Ab;
              but, AB is A, and Ab is A;
                    therefore, X is A.

We have also, X is A;
                    but, A is AB or Ab,
                  therefore, X is AB or Ab.

(8) *If in a series of alternatives occurring either in the subject or in the predicate of a proposition, the contradictory of any given alternative appears combined with other terms in other alternatives it may be omitted from the latter without altering the force of the assertion.*

Thus, "A or aB" may be replaced by "A or B," and *vice versa.*

For,   given X is A or aB;

since aB is B,

it follows that X is A or B.

And, conversely, given X is A or B;

since B is AB or aB,

it follows that X is A or AB or aB;

but, AB is A,

therefore, X is A or aB.

Thus, we may not merely infer

"X is A or B" from "X is A or aB";

but we may do this without any loss of force.

Again, given No X is either A or aB;

since AB is A,

it follows that X is not AB;

therefore, No X is either AB or aB;

but B is either AB or aB,

therefore, X is not B;

and therefore, No X is either A or B.

In this case the passage back from

No X is either A or B,

to No X is either A or aB,

is still more obvious.

294. The Introduction of fresh Terms into Complex Propositions without affecting the force of the assertion.

This is in itself the reverse of simplification, but it is in some cases a necessary introduction to a process of simplification.

It is clear that we have a case corresponding to each of the cases just discussed. Wherever we may obtain a proposition equivalent to a given proposition by dropping a

term, we may correspondingly obtain a proposition equivalent to a given proposition by the introduction of a fresh term. The proof of each separate case has been given in establishing the various equivalences in the preceding section.

The following are the more important cases :

(3) *In a universal negative proposition any term that appears in every alternative in the subject may be combined with any alternative in the predicate; similarly, any term that appears in every alternative in the predicate may be combined with any alternative in the subject; and in neither case will the force of the original statement be affected.*

(4) *In an affirmative proposition any term that appears in every alternative in the subject may be combined with any alternative in the predicate without affecting the force of the statement.*

(7) *If any term either in the subject or in the predicate of a proposition is developed by means of the law of excluded middle* (cf. section 289) *we obtain an equivalent proposition.*

(8) *In a series of alternatives occurring either in the subject or in the predicate of a proposition, the contradictory of any given alternative may be combined with other alternatives without altering the force of the assertion.* For example, "*A* or *aB*" may be substituted for "*A* or *B*."

295. Types of Equivalent Propositions, as established in the two preceding sections.

$$(1) \quad \begin{cases} X \text{ is } ABB; \\ X \text{ is } AB. \end{cases}$$

$$(2) \quad \begin{cases} X \text{ is } AB \text{ or } AB; \\ X \text{ is } AB. \end{cases}$$

(3) $\begin{cases} \text{No } AX \text{ is } AB \text{ or } ACD; \\ \text{No } AX \text{ is } B \text{ or } CD; \\ \text{No } X \text{ is } AB \text{ or } ACD. \end{cases}$

(4) $\begin{cases} A \text{ is } AB \text{ or } ACD; \\ A \text{ is } B \text{ or } CD. \end{cases}$

(5) $\begin{cases} X \text{ is } A \text{ or } B \text{ or } BC; \\ X \text{ is } A \text{ or } B. \end{cases}$

(6) $\begin{cases} X \text{ is } A \text{ or } Bb; \\ X \text{ is } A. \end{cases}$

(7) $\begin{cases} X \text{ is } ABC \text{ or } ABc; \\ X \text{ is } AB. \end{cases}$

(8) $\begin{cases} X \text{ is } A \text{ or } aB; \\ X \text{ is } A \text{ or } B. \end{cases}$
$\begin{cases} \text{No } X \text{ is } A \text{ or } aB; \\ \text{No } X \text{ is } A \text{ or } B. \end{cases}$

296. Shew that "a or b or cd" is the contradictory of "ABC or ABD."

A proof of this was promised in section 288. Applying the rule laid down in that section, we have for the contradictory of the given term,—

(a or b or c) and at the same time (a or b or d).

We have therefore to combine each of the first set of alternatives with each of the second set. This yields

aa or ab or ad or ab or bb or bd or ac or bc or cd.

But we have shewn that aa may be replaced by a, and bb by b; that since ab, ad, and ac are subdivisions of a, and a is one of the alternatives, they may be omitted; and similarly with bd and bc, in consequence of their relation to b.

The contradictory is therefore reduced to a or b or cd.

297. State the contradictories of the following terms in their simplest forms :—

$$AB \text{ or } BC \text{ or } CD,$$
$$AB \text{ or } bC \text{ or } cD,$$
$$ab \text{ or } BC \text{ or } cd,$$
$$AB \text{ or } bC \text{ or } Cd.$$

298. Shew that the two following propositions are equivalent to each other :—

(1) X is BC or bD or CD,

(2) X is BC or bD.

299. Shew the equivalence between the propositions,—

(1) XY is either aB or aC or bC or aE or bE or Ad or Ae or bd or be or cd or ce ;

(2) XY is either a or b or d or e.

(2) follows immediately from (1); but it is important to notice also that nothing is lost in this inference, *i.e.*, that we may pass back from (2) to (1). This follows immediately from the principles established in sections 293—295.

The steps may be shewn at length as follows, (the numbers indicating the processes made use of as described in the above sections). A comma is here placed between the different alternatives.

$$a, \ b, \ d, \ e ;$$

$a, \ b, \ d, \ cd, \ e, \ ce$;	(5)
$a, \ b, \ Ad, \ cd, \ Ae, \ ce$;	(8)
$a, \ aC, \ aE, \ b, \ Ad, \ cd, \ Ae, \ ce$;	(5)
$aB, \ aC, \ aE, \ b, \ Ad, \ cd, \ Ae, \ ce$;	(8)
$aB, \ aC, \ aE, \ b, \ bC, \ bd, \ Ad, \ cd, \ Ae, \ ce$;	(5)
$aB, \ aC, \ aE, \ bE, \ be, \ bC, \ bd, \ Ad, \ cd, \ Ae, \ ce.$	(7)

300. Shew the equivalence between the two propositions :—

(1) XY is aBC or aCD or aBe or aDe or AcD or abD or bcD or aDE or cDE ;

(2) XY is aBC or aD or cD or aBe.

301. Shew the equivalence between each pair of the following propositions :—

(i) X is either AB or AC or BC or abc or aB or C ;

X is either a or B or C.

(ii) X is either aBC or aBd or acd or bcd or ABd or Acd or abd or aCd or BCd ;

X is either Bd or cd or ad or aBC or aCd.

(iii) X is either Pqr or pQs or prs or qrs or pq or pS or qR ;

X is either p or q.

302. Shew the equivalence between the two propositions :—

No XY is either aB or aC or bC or aE or bE or Ad or Ae or bd or be or cd or ce ;

No XY is either a or b or d or e.

Here it is immediately obvious that the first proposition is inferrible from the second. It may be shewn that the second is inferrible from the first by a kind of converse process to that employed in section 299.

303. Shew the equivalence between the two propositions :—

No XY is aBC or aCD or aBe or aDe or AcD or abD or bcD or aDE or cDE ;

No XY is aBC or aD or cD or aBe.

No difficulty will be found with this example if the student notices that "neither AcD nor aD" may be reduced to "neither cD nor. aD," since cD must be either AcD or aD.

304. Shew the equivalence between the following pairs of propositions :—

(i) No X is either AB or AC or BC or abc or aB or C;

No X is either a or B or C.

(ii) No X is either aBC or aBd or acd or bcd or ABd or Acd or abd or aCd or BCd;

No X is either Bd or cd or ad or aBC or aCd.

(iii) No X is either Pqr or pQs or prs or qrs or pq or pS or qR;

No X is either p or q.

305. Simplify the propositions :—

(1) X is Ab or aC or BCd or Bc or bD or CD.

(2) X is ACD or Ac or Ad or aB or bCD.

306. Inference by the Omission of Terms, or by the Introduction of fresh Terms in Complex Propositions, the inferred proposition, however, not being equivalent to the original proposition.

(1) A fresh term may always be introduced into the subject of a proposition, (though the force of the proposition is thereby weakened); but no term may ever be omitted from the subject of a proposition, (except in the case of a negative proposition where the same term appears also in the predicate as shewn in section 293).

It is clear that whatever may be affirmed (or denied) of A may be affirmed (or denied) of AB; in other words, whatever is true of A is true of that which is both A and B.

But we cannot on the other hand pass back from affirming (or denying) anything of AB to affirming (or denying) the same thing of A.

(2) A fresh term may always be introduced into the predicate of a negative proposition; but not into the predicate of an affirmative proposition, unless it already appears in the subject[1].

(3) A term may always be dropped from the predicate of an affirmative proposition; but not from the predicate of a negative proposition, unless it also appears in the subject[1].

It is clear that if No A is B, then No A is both B and C; but not *vice versa*, since although No A is both B and C, All A might be B and not-C. Again, it is clear that if All A is both B and C, then All A is B; but it does not follow that if All A is B, therefore All A is both B and C.

307. If no A is bc or Cd, it follows that no A is bd.

308. Interpretation of propositions of the forms No AB is B, AB is a, AB is Cc.

Propositions of the above kind may easily result as a consequence of the manipulation of complex propositions; but they involve a contradiction in terms and are in direct contravention of the fundamental laws of thought. They must be interpreted as affirming the non-existence of the

---

[1] Cf. sections 293, 294.

subject of the proposition. Thus, AB is a is to be interpreted No A is B, or A is b.

This must be taken in connection with the discussion in section 106. The view was there adopted that no universal proposition implies the existence of its subject; but if it is affirmative it denies the existence of anything that is the subject and is not the predicate. Thus AB is a denies the existence of anything that is at the same time AB and not-a, *i.e.*, A. But AB is AB and A. The existence of AB is therefore denied.

Similarly, a universal negative proposition denies the existence of anything that is both subject and predicate. No AB is B denies the existence of ABB, *i.e.*, of AB as before.

AB is Cc affirms that AB is something that is non-existent, and therefore that it is itself non-existent.

If the view were adopted that a proposition *does* imply the existence of its subject, then if propositions of the above form were obtained, we should be thrown back on the alternative that some inconsistency had already found place in the premisses.

CHAPTER III.

309. If from No A is BC, I infer that No B is AC, what is the nature of the inference? [v.]

This inference is of the nature of Conversion, but three terms being involved, it is necessarily more complex than those cases of conversion which have been previously considered. It may be simply analysed as follows,—

No A is both B and C,

therefore, Nothing is at the same time A, B, and C,

therefore, No B is both A and C.

The reasoning may also be resolved into a series of ordinary conversions :—

No A is BC,

therefore (by conversion), No BC is A,

i.e., within the sphere of C, No B is A,

therefore (by conversion), within the sphere of C, No A is B,

i.e., No AC is B,

therefore (by conversion), No B is AC.

Or, it may be treated thus,—

$$\text{No } A \text{ is } BC,$$
$$\text{therefore } a\ fortiori, \text{ No } AC \text{ is } BC^{1},$$

therefore, No AC is B, (for if any AC were B, it would necessarily be $BC)^{1}$,

$$\text{therefore (by conversion), No } B \text{ is } AC.$$

310. The application of the term *Conversion* to propositions containing more than two terms.

Generalising, we may say that we have a process of Conversion when from a given proposition we infer a new one in which a term that appeared in the predicate of the original proposition now appears in the subject, or *vice versa*.

If a complex proposition, (by which I here mean a proposition containing more than two terms), *is a universal negative, any term may be transferred from subject to predicate or* vice versa *without affecting the force of the assertion.*

We have just shewn how from

$$\text{No } A \text{ is } BC,$$

we may obtain by conversion

$$\text{No } B \text{ is } AC.$$

Similarly, we may infer

$$\text{No } C \text{ is } AB,$$
$$\text{No } AB \text{ is } C,$$
$$\text{No } AC \text{ is } B,$$
$$\text{No } BC \text{ is } A.$$

The proposition might also be written,—

$$\text{There is no } ABC,$$
or, Nothing is at the same time A, B and C.

---

[1] Cf. sections 293, 294.

The application of the process of Conversion to affirmative propositions is of less importance; since the converse of an affirmative proposition whether simple or complex is always particular. Particular propositions are not in themselves of great value; and, as shewn in Part II., chapter VIII., they may involve us in troublesome questions with regard to " existence."

In dealing with complex propositions, it is especially desirable, or even essential, to keep clear of the implication of the existence of the subject of the proposition. I proceed always on the hypothesis that a universal proposition in no case does more than negative the existence of certain combinations. Thus, No A is BCD negatives $ABCD$; All AB is CD negatives ABc and ABd, (as usual denoting not-C by c and not-D by d).

It is worth while pointing out that from All A is BC we may obtain by conversion Some B is AC, and Some C is AB; but in complicated inferences we shall hardly ever have occasion to convert affirmative propositions in this way. We shall find however that to counterbalance this, the process of contraposition is particularly valuable in its application to complex universal affirmative propositions.

311. Shew clearly that if No De is ABc, then No $ABcD$ is e; if No c is Bdk, then No Bd is ck; if No $AbDF$ is K, then No $AbcDE$ is FK; if ABC is EF, then $ABCG$ is BE; if No $AbDE$ is bCE, then No $CDEF$ is AbH.

CHAPTER IV.

THE OBVERSION OF COMPLEX PROPOSITIONS.

312. The Obversion of Propositions containing more than two terms.

The doctrine of Obversion is immediately applicable to Complex Propositions; and we require no modification of our former definition of Obversion. From any given proposition we may infer a new one by changing its quality and taking as a new predicate the contradictory of the original predicate. The proposition thus obtained is called the obverse of the original proposition.

The only difficulty connected with the obversion of complex propositions consists in finding the contradictory of a complex term. We have, however, in section 288, given a simple rule for finding the contradictory of any complex term :—*For each simple term involved, substitute its contradictory ; write* and *for* or, *and* or *for* and.

Applying this rule to "AB or ab," we have "(a or b) and (A or B)," *i.e.,* "Aa or Ab or aB or Bb"; but since Aa and Bb involve self-contradiction, they may, as shewn in section 293, be omitted. The obverse, therefore, of "All X is AB or ab" is "No X is Ab or aB."

313. Find the obverse of each of the following propositions :—

    (1) *A* is *BC*,

    (2) *A* is *BC* or *DE*,

    (3) No *A* is *BcE* or *BCF*,

    (4) No *A* is *B* or *bcDEf* or *bcdEF*.

(1) "*A* is *BC*" gives at once " No *A* is *b* or *c*."

(2) "*A* is *BC* or *DE*" gives "No *A* is (*b* or *c*) and at the same time (*d* or *e*)." As already pointed out, it may be necessary to use brackets in this way to avoid ambiguity. Without brackets, however, and avoiding all chance of ambiguity, we may write the above,—"No *A* is *bd* or *be* or *cd* or *ce*." The student should make it very clear to himself that these two forms are really equivalent.

(3) " No *A* is *BcE* or *BCF*." Here by the application of the general rule we have as the contradictory of the predicate,—"(*b* or *C* or *e*) and at the same time (*b* or *c* or *f*)." What is "both *b* and *b*" is of course "*b*," and we have no more information about a thing if we are told that it is "both *b* and *b*" than if we are told that it is simply "*b*"; it has also been already pointed out that such a term as *Cc* must represent what is non-existent, and therefore when it is given as one among several alternatives it may be neglected; again as shewn in section 293 such an expression as " *b* or *bc* or *bf* or *Cf*" may be simplified to " *b* or *Cf*." Remembering these three points, we find that " (*b* or *C* or *e*) and (*b* or *c* or *f*)" may be written "*b* or *Cf* or *ce* or *ef*." For the obverse of the given proposition, we have, therefore, "*A* is *b* or *Cf* or *ce* or *ef*."

(4) " No *A* is *B* or *bcDEf* or *bcdEF*." The obverse is,—"*A* is *b* and (*B* or *C* or *d* or *e* or *F*) and (*B* or *C* or *D* or *e* or *f*)"; *i.e.*, "*A* is *bC* or *bDF* or *be* or *bdf*."

314. Find the obverse of each of the following propositions :—

(1) Nothing is X, Y or Z ;

(2) X is Ab or aC ;

(3) W is XZ or Yz or YZ or Xy or xZ ;

(4) Ab is $CDEf$ or Cd or cDf or cdE ;

(5) No De is ABC or Abc ;

(6) No A is Cd or cD or bcd.

315. No citizen is at once a voter, a householder and a lodger ; nor is there any citizen who is neither of the three.

Every citizen is either a voter but not a householder, or a householder and not a lodger, or a lodger without a vote.

Are these statements precisely equivalent ? [v.]

It may be shewn that each of these statements is the logical obverse of the other. They are therefore precisely equivalent.

Let V = voter, v = not voter;

    H = householder, h = not householder;

    L = lodger, l = not lodger.

The first of the given statements is

No Citizen is VHL or vhl ;

therefore (by obversion), Every citizen is either v or h or l and is also either V or H or L ;

therefore (combining these possibilities), Every citizen is either Hv or Lv or Vh or Lh or Vl or Hl.

But (by the law of Excluded Middle), Hv is either HLv or Hlv ;

therefore,   *Hv* is *Lv* or *Hl.*

   Similarly,   *Lh* is *Vh* or *Lv;*

and   *Vl* is *Hl* or *Vh.*

Therefore,   Every citizen is *Vh* or *Hl* or *Lv,*

which is the second of the given statements.

   Again, starting from this second statement, it follows
(by obversion) that No citizen is at the same time *v* or *H,*
h or *L, l* or *V;*

therefore, No citizen is *vh* or *vL* or *HL,* and at the same
time *l* or *V;*

therefore,   No citizen is *vhl* or *VHL,*

which brings us back to the first of the given statements.

   316. Shew that the two following propositions
are equivalent :—

  · No *X* is *A* or *BC* or *BD* or *DE,*

   X is *aBcd* or *abDe* or *abd.*

CHAPTER V.

317. The application of the term Contraposition to propositions containing more than two terms.

According to our original definition, we contraposit a proposition when we infer from it a new proposition which has the contradictory of the old predicate for its subject and the old subject for its predicate.

Thus, " No not-B is A" is the contrapositive of "All A is B"; "All not-B is not-A" is its obverted contrapositive. Similarly, the contrapositive of "A is B or C" would be "No bc is A", the obverted contrapositive "bc is a". The contrapositive of "A is BC" would be "No b or c is A." It will be observed, therefore, that the old rule for obtaining the contrapositive still applies, namely,—first obvert the given proposition, and then convert it.

The contrapositive of a negative proposition is as before particular, and may be practically neglected.

The following simple rule may then be given for obtaining the *obverted* contrapositive of a universal affirmative proposition :— *Take as a new subject the contradictory of the old predicate, and as a new predicate the contradictory of the*

old subject, the proposition still remaining affirmative.   For
example,—

    A is *BC*, therefore, whatever is *b* or *c* is *a*.

    A is *B* or *C*, therefore, *bc* is *a*.

    A is *BC* or *E*, therefore, whatever is *be* or *ce* is *a*.

So far I have been discussing what may be called the
full contrapositive of a complex proposition ; and starting
with a universal affirmative we can pass back from such a
contrapositive to the original proposition.   In other words,
any universal affirmative proposition and its full contra-
positive are *equivalent* propositions.

In relation to complex propositions, however, we shall
find it convenient to give the term Contraposition an ex-
tended meaning.   We may say that *we have a process of
Contraposition when from a given proposition we infer a new
one in which the contradictory of a term that appeared in the
predicate of the original proposition now appears in the subject,
or the contradictory of a term that appeared in the subject of
the original proposition now appears in the predicate.*

We may distinguish *four* operations which will be in-
cluded under this definition :

(1) The operation of obtaining the *full* contrapositive
of a given proposition, as above described and defined.

(2) From "*A* is *BC* or *E*", we may infer "whatever is
be or *ce* is *a*"; but in a given application it may be sufficient
for us to know that "*be* is *a*", and although this is not the
full contrapositive of the original proposition, we may
regard it as immediately obtained from the original pro-
position by a process of contraposition.

With reference to this case, the following general rule
may be given,—*If one or other of a series of alternatives is*

*predicated of a subject, the contradictory of this subject may be
predicated of any term that is incompatible with all these alter-
natives.* Thus, if "A is PqR or pRS", we may infer that "ps
is a"; since ps is neither PqR nor pRS, and therefore what
is ps cannot be A. But we have not here the full contra-
positive of the given proposition, and we could not pass
back from "ps is a" to "A is PqR or pRS", but only to
"A is P or S."

(3) From the proposition "A is B or C", it follows that
if A is not B, it is C; but this is expressed by the proposition
"Ab is C", and the contradictory of a term that originally
appeared in the predicate now appears in the subject,—
i.e., according to the above definition we have a process of
contraposition. This process might also be described as *the
omission of one or more of a series of alternatives in the predi-
cate by a further particularisation of the subject.*

With reference to this case, the following general rule
may be given,—*If any term X is combined with every al-
ternative in the subject of a proposition, every alternative in
the predicate which contains the contradictory of this term may
be omitted*[1]. Thus, from

   Whatever is A or B is C or DX or Ex,
we may obviously infer

   Whatever is AX or BX is C or D.

[1] This may sometimes result in the disappearance of all the alterna-
tives; and the meaning of such result is that we now have a non-
existent subject.

   Thus, given

              P is $ABCD$ or $Abcd$ or $aBCd$,

if we particularise the subject by making it PbC, we find that all the
alternatives in the predicate disappear. The interpretation is that the
class PbC is non-existent, *i.e.,* No P is bC; a conclusion which of
course might also have been obtained directly from the given proposi-
tion.

(4) The last operation to which reference is made above is the reverse of that which we have just discussed. From the proposition "AB is C", we may infer "A is b or C". This may be described as *a generalisation of the subject by the addition of one or more alternatives in the predicate.* But it is also clear that it comes under the extended meaning that we have given to the term Contraposition.

To meet this case, the following general rule may be given,—*Any term that appears in the subject of a proposition may be dropped therefrom, if its contradictory is at the same time added as an additional alternative in the predicate.*

The following may be taken as typical examples of all the operations that we now include under contraposition:—

$$AB \text{ is } CD \text{ or } de;$$

therefore, *first,* anything that is either cD or cE or dE is a or b, (the *full* contrapositive, obverted, according to our original definition);

secondly,	cE is a or b;
thirdly,	ABD is C;
fourthly,	A is b or CD or de.

Combinations of the third and fourth operations give

$$AcD \text{ is } b;$$
$$Ad \text{ is } b \text{ or } e;$$
$$\&c.$$

The first of the above being called the *Full* Contrapositive of the given proposition, the remaining inferences may be called *Partial* Contrapositives, according to our extended definition of contraposition. In each case, some term disappears from the subject or from the predicate of the original proposition, and is replaced by its contradictory in the predicate or the subject accordingly. Only in the full contrapositive, however, is *every* term thus transposed.

I do not think that any confusion need result from the nomenclature now proposed, since the extended use of the term Contraposition can be applied only to complex propositions. There is still only one kind of Contraposition possible in the case of the categorical proposition containing but two terms.

The great importance of Contraposition as we are now dealing with it in connection with complex propositions is that by its means, *given a universal affirmative proposition of any complexity, we may obtain separate information with regard to any term that appears in the subject, or with regard to the contradictory of any term that appears in the predicate, or with regard to any combination of these terms.* Thus, given "XY is P or Qr", by the process described as the generalisation of the subject, we have

$$X \text{ is } y \text{ or } P \text{ or } Qr.$$

The particularisation of the subject gives

$$XYp \text{ is } Qr,$$
$$XYq \text{ is } P,$$
$$\&c.;$$

and by the combination of these processes, we have

$$Xp \text{ is } y \text{ or } Qr;$$
$$\&c.$$

Again, the full contrapositive of the original proposition is

$$\text{Whatever is } pq \text{ or } pR \text{ is } x \text{ or } y;$$

from which we have

$$p \text{ is } x \text{ or } y \text{ or } Qr,$$
$$q \text{ is } x \text{ or } y \text{ or } P,$$
$$\&c.$$

318. Given "All D that is either B or C is A," shew that "Everything that is not-A is either not-B and not-C or else it is not-D."   [De Morgan.]

This example and the five following examples are adapted from De Morgan, *Syllabus*, p. 42. They are also given by Jevons, *Studies*, p. 241, in connection with his Equational Logic. They are all simple exercises in Contraposition.

We have, What is either BD or CD is A,

therefore, a is (b or d) and (c or d),

therefore, a is bc or d.

319. Given "All A is either BC or BD," shew that "All that is not-B and not-C is not-A" and "All that is not-D is not-A."   [De Morgan.]

320. If A is either BC or D, and if whatever is BC or D is A, shew that whatever is not-A is not-D and also either not-B or not-C, and whatever is not-D and at the same time not-B or not-C is not-A.

[De Morgan.]

321. If whatever is B or CD or CE is A, what do we know about not-A ?   [De Morgan.]

322. If whatever is either B or C and at the same time either D or E is A, what do we know about not-A ?   [De Morgan.]

323. If that which is A or BC and is also D or EF is X, what is all that we know about not-X ?

[De Morgan.]

What is (A or BC) and (D or EF) is X;

therefore (by contraposition), x is ab or ac or de or df.

[There is apparently a misprint in Jevons's transcription of this example (*Studies*, p. 241). He uses different letters, but his implied solution is "*x* is either *ab* or *c* and it is also either *de* or *f*". I cannot see how this is obtainable.]

324. To say that whatever is devoid of the properties of *A* must have those either of *B* or of *D*, or else be devoid of those of *C*, is the same as to say that what is devoid of the properties of *B* and *D*, but possesses those of *C*, must have *A*. Prove this.

[Jevons, *Studies*, p. 239.]

325. Shew that "*C* is *Ab* or *aB*" is equivalent to the two propositions "*AB* is *c*" and "*ab* is *c*".

[Jevons, *Studies*, p. 239.]

326. Prove the equivalence of the following assertions :—

(1) Every gem is either rich or rare.

(2) Every gem which is not rich is rare.

(3) Every gem which is not rare is rich.

(4) Everything which is neither rich nor rare is not a gem.   [Jevons, *Studies*, p. 229.]

327. If that which is devoid of heat and devoid of visible motion is devoid of energy, it follows that what is devoid of visible motion but possesses energy cannot be devoid of heat. [Jevons, *Studies*, p. 199.]

328. If the relations *A* and *B* combine into *C*, it is clear that *A* without *C* following means that there is not *B*, and that *B* without *C* following means that there is not *A*.   [De Morgan.]

329. Any one who wishes to test himself and his friends upon the question whether analysis of the forms of enunciation would be useful or not, may try himself and them on the following question :—

Is either of the following propositions true, and if either, which ?

(1) All Englishmen who do not take snuff are to be found among Europeans who do not use tobacco.

(2) All Englishmen who do not use tobacco are to be found among Europeans who do not take snuff.

Required immediate answer and demonstration.

[De Morgan.]

330. Is the student of logic, generally speaking, prepared *rapidly* to analyse the two following propositions, and to say whether or no they must be identical, if the identity of synonyms be granted ?

(1) The suspicion of a nation is easily excited, as well against its more civilised as against its more warlike neighbours, and such suspicion is with difficulty removed.

(2) When we see a nation either backward to suspect its neighbour, or apt to be satisfied by explanations, we may rely upon it that the neighbour is neither the more civilised nor the more warlike of the two.   [De Morgan.]

331. Infer all that you possibly can by way of Contraposition or otherwise, from the assertion, All *A* that is neither *B* nor *C* is *X*.   [R.]

The given proposition may be written *Abc* is *X*; and taking it as it stands, the converse is Some *X* is *Abc*, and the contrapositive (obverted) *x* is *a* or *B* or *C*. We may also get a number of other propositions with which we may proceed in the same way; *e.g.*,—

$$Abx \text{ is } C,$$
$$Acx \text{ is } B, \&c.$$

Confining ourselves however to such universal propositions as can be obtained, the problem may be solved generally as follows :—

The given proposition may be thrown into the form,—

Nothing is at the same time *A*, *b*, *c* and *x*;

and we see that it is symmetrical with regard to the terms *A*, *b*, *c*, *x*. We are sure then that anything that is true of *A* is true *mutatis mutandis* of *b*, *c* and *x*, that anything that is true of *Ab* is true *mutatis mutandis* of any pair of the terms, and similarly for combinations three and three together.

We have at once the four symmetrical propositions,—

$$A \text{ is } B \text{ or } C \text{ or } X; \quad (1)$$
$$b \text{ is } a \text{ or } C \text{ or } X; \quad (2)$$
$$c \text{ is } a \text{ or } B \text{ or } X; \quad (3)$$
$$x \text{ is } a \text{ or } B \text{ or } C. \quad (4)$$

Then from (1) we have

$$Ab \text{ is } C \text{ or } X; \quad (i)$$

and the five corresponding propositions are,—

$$Ac \text{ is } B \text{ or } X; \quad (ii)$$
$$Ax \text{ is } B \text{ or } C; \quad (iii)$$
$$bc \text{ is } a \text{ or } X; \quad (iv)$$
$$bx \text{ is } a \text{ or } C; \quad (v)$$
$$cx \text{ is } a \text{ or } B. \quad (vi)$$

Again from (i),—

Abc is *X*, (which is the original proposition),   (α)
and we have, similarly,—

$$
\begin{array}{ll}
\textit{Abx is C}; & (\beta) \\
\textit{Acx is B}; & (\gamma) \\
\textit{bcx is a}. & (\delta)
\end{array}
$$

It should be noted that the following are pairs of contrapositives,—

(1) (δ), (2) (γ), (3) (β), (4) (α), (i) (vi), (ii) (v), (iii) (iv).

332. Find the *full* contrapositive of each of the following propositions :—

A is *BCDe* or *bcDe* ;

AB is *CD* or *cDE* or *de* ;

Whatever is *AB* or *bC* is *aCd* or *Acd*;

Where *A* is present along with either *B* or *C*, *D* is present and *C* absent or *D* and *E* are both absent ;

Whatever is *ABC* or *abc* is *DEF* or *def*.

333. Compare the logical force of the following propositions :—

(1) All voters who are not lodgers are householders who pay rates ;

(2) No one who is not a lodger and who does not pay rates is a voter ;

(3) A voter who is a householder is not a lodger;

(4) A householder who does not pay rates is not a voter ;

(5) All who pay rates or are householders are voters ;

(6) Anyone who is not a householder or who being a householder does not pay rates is either not a voter or else he is a lodger;

(7) All who have a vote pay rates;

(8) Anyone who has no vote is either not a rate-payer or not a householder.

334. If A is either B or C, shew that what is not B is either C or not A.

335. What is the difference between the assertion that A is BC and the pair of assertions that b is a, and c is b?   [Jevons, *Studies*, p. 239.]

336. If A unless it is B is either CD or EF, shew that not-C is either not-A or B or EF.

337. Establish the following,—

(i) Where B is absent, either A and C are both present or A and D are both absent; therefore, where C is absent, either B is present or D is absent.

(ii) Where A is present and also either B or E, either C is present and D absent or C is absent and D present; therefore, where C and D are either both present or both absent, either A is absent or B and E are both absent.

(iii) Where A is present, either B and C are both present or C is present D being absent or C is present F being absent or H is present; therefore, where C is absent, A cannot be present H being absent.

338. Among plane figures the circle is the only curve of equal curvature. Shew that this is the same

as to assert that a plane figure must either be a curve of equal curvature, in which case it is also a circle, or else, not a circle and then not a curve of equal curvature.   [Jevons, *Studies*, p. 235.]

Let P = plane figure,

C = circle,

E = curve of equal curvature.

"Among plane figures the circle is the only curve of equal curvature," may be expressed by PC is CE, and Pc is ce. "A plane figure must either be a curve of equal curvature, in which case it is also a circle, or else, not a circle and then not a curve of equal curvature," becomes P is CE or ce. It is immediately obvious that the two statements are equivalent.

339. "Similar figures consist of all figures whose corresponding angles are equal and whose sides are proportional." Give *all* the propositions involving not more terms, which can be inferred from the above. Give also one proposition equivalent to it.   [L.]

Let P = similar figures,

Q = figures whose corresponding angles are equal,

R = figures whose sides are proportional.

The given statement may be resolved into the two propositions,—

> All P is QR,
> All QR is P.

From these, by contraposition, we may infer,—

> p is q or r;
> Q is PR or pr;
> q is p;

R is PQ or pq;
r is p;
PQ is R;
PR is Q;
pQ is r;
pR is q;
Qr is p;
qR is p;
qr is p.

Pq and Pr represent non-existent classes; and we have no information with regard to pq and pr.

This I think affords a complete solution of the first part of the question. We may obtain a statement equivalent to the given statement, by taking the full contrapositives of the two propositions into which we resolved it and then combining them. Thus,

P is QR, $= q$ or r is p,
QR is P, $= p$ is q or r;

and these are combined in the statement that p consists of all things that are q or r. "Figures that are not similar consist of all figures whose corresponding angles are not equal or whose sides are not proportional."

340. Given A is BC, what, if anything, do you know concerning the classes AB, Ab, AC, Ac, a, aB, ab, aC, ac, B, BC, Bc, b, bC, bc, C, c?

A is BC,

therefore, by conversion, Some BC is A.   (1)

By contraposition, we may obtain the two propositions,

No b is A,   (2)

No c is A.   (3)

Then by once more obverting and converting,

$$\left. \begin{array}{l} \text{Some } a \text{ is } b, \\ \text{Some } a \text{ is } c. \end{array} \right\} \qquad (4)$$

We cannot combine these into " Some a is bc " since we do not know that the same a is referred to in both cases.

The other forms which can be obtained are in reality only weakened forms of one or other of the above.

By (3) No c is A,

i. e., Nothing is Ac,   (5)

therefore (*a fortiori*), No B is Ac.   (6)

Similarly, by (2),   No b is A,

i. e., Nothing is Ab,   (7)

therefore (*a fortiori*), No C is Ab.   (8)

Again, by obversion of the original proposition,

No A is c,

therefore (*a fortiori*), No AB is c.   (9)

Similarly,   No AC is b.   (10)

Also from (2),   No bC is A,   (11)

and No bc is A.   (12)

Similarly, from (3), No Bc is A.   (13)

We cannot obtain any information with regard to the remaining classes aB, ab, aC, ac.

[As already indicated, I should consider that (1) and (4) involve assumptions with regard to " existence." Without any such assumptions, however, we can obtain all the remaining inferences. We may regard (4) as obtained by inversion of the original proposition. Cf. section 348.]

341. Assuming that armed steam-vessels consist of the armed vessels of the Mediterranean and the steam-vessels not of the Mediterranean, inquire whether we can thence infer the following results:—

(1) There are no armed vessels except steam-vessels in the Mediterranean.

(2) All unarmed steam-vessels are in the Mediterranean.

(3) All steam-vessels not of the Mediterranean are armed.

(4) The vessels of the Mediterranean consist of all unarmed steam-vessels, any number of armed steam-vessels, and any number of unarmed vessels without steam.   [Jevons, *Studies*, p. 231, from Boole.]

342. If AB is either Cd or cDe, and also either eF or H, and if the same is true of BH, what do we know of that which is E?

We have given,—

What is AB or BH is (Cd or cDe) and (eF or H);
therefore, What is AB or BH is $CdeF$ or $cDeF$ or CdH or $cDeH$;

therefore, What is ABE or BHE is CdH;
therefore, E is CdH or b or ah.

343. If A that is B is either P or Q and also either R or S, and if the same is true of A that is both C and D, what is all that we know about that which is neither P nor S?

344. Given that whatever is PQ or AP is bCD or $abdE$ or $aBCdE$ or $Abcd$, shew that,—

(1) abP is CD or dE or q ;
(2) DP is bC or aq ;
(3)   Whatever is B or Cd or cD is a or p ;
(4) B is C or p or aq ;

(5) Cd is a or p ;

(6) AB is p ;

(7)   If ae is c or d it is p or q ;

(8)   If BP is c or D or e it is aq.

345. Given A is BC or BDE or BDF, infer descriptions of the following terms Ace, Acf, $ABcD$.

[Jevons, *Studies,* pp. 237, 238.]

In accordance with rules already laid down, we have immediately,—

Ace is BDF;

Acf is BDE ;

$ABcD$ is E or F.

346. Given b is CDe or Acd or $adeF$ or $acdEF$, infer descriptions of A, bf, CD, cD, de.

347. Given that PQr is ABc or abD or $aCDE$ or $BCdeF$ or $bCdf$ or $CDEF$ or def, what is all that you know concerning the classes,—

A, a, B, b, C, c, D, d, E, e, F, f ?

348. The Inversion of Complex Propositions.

We might define *Inversion* in connection with complex propositions as a process by which from a given proposition we infer a new one in which some term in the subject is replaced by its contradictory. I have not, however, thought it worth while to give any detailed discussion of inversion here, because this process always results in particular propositions ; these are of small importance at the best, and they involve assumptions concerning the existence of their subjects, which are so inconvenient when these subjects are very complex, that they are best neglected altogether, unless a very special treatment is accorded to them.

349. Summary of the results obtainable by Conversion, Obversion, and Contraposition.

(1) By *Conversion* of a universal negative we can obtain separate information with regard to any term that appears either in the subject or in the predicate, or with regard to any combination of these terms.

For example,   No *AB* is *CD* ;

therefore, No *A* is *BCD*,

No *C* is *ABD*,

No *BD* is *AC*.

(2) By *Obversion* we can change any proposition from the affirmative to the negative form, or *vice versa*.

For example, *AB* is *CD* or *EF*; therefore, No *AB* is *ce* or *cf* or *de* or *df*.

No *P* is *QR* ;

therefore, *P* is *q* or *r*.

(3) By *Contraposition* of a universal affirmative we can obtain information with regard to any term that appears in the subject, or with regard to the contradictory of any term that appears in the predicate, or with regard to any combination of these terms.

For example,   *AB* is *CD* or *EF*;

therefore, *A* is *b* or *CD* or *EF*,

c is *a* or *b* or *EF*,

Be is *a* or *CD*,

ce is *a* or *b*,

Adf is *b*,

&c.

CHAPTER VI.

THE COMBINATION OF COMPLEX PROPOSITIONS.

350. The Combination of Universal Affirmative Propositions the subjects of which do not contain Contradictories.

$$X \text{ is } P_1 \text{ or } P_2 \text{ or } \ldots \text{ or } P_m,$$
$$Y \text{ is } Q_1 \text{ or } Q_2 \text{ or } \ldots \text{ or } Q_n,$$

may be taken as types of two such propositions.

By combining them we have

$$XY \text{ is } (P_1 \text{ or } P_2 \text{ or } \ldots \text{ or } P_m), \text{ and also}$$
$$(Q_1 \text{ or } Q_2 \text{ or } \ldots \text{ or } Q_n);$$
$$i.e., XY \text{ is } P_1Q_1 \text{ or } P_1Q_2 \text{ or } \ldots \text{ or } P_1Q_n$$
$$\text{ or } P_2Q_1 \text{ or } P_2Q_2 \text{ or } \ldots \text{ or } P_2Q_n$$
$$\text{ or } \ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$\text{ or } P_mQ_1 \text{ or } P_mQ_2 \text{ or } \ldots \text{ or } P_mQ_n.$$

If the subject of both the original propositions had been X, then of course we should have

$$X \text{ is } P_1Q_1 \text{ or } P_1Q_2 \text{ or } \ldots\ldots \&c.$$

In this case, the new proposition is equivalent to the two propositions with which we started, *i.e.*, we could pass back

from it to them. But when the subjects of the original propositions are not the same, the new proposition is not equivalent to them.

In combining propositions, the student should never lose an opportunity of simplifying his results; and such opportunities will be found to be of continual recurrence.

The following are examples :

$$(1) \quad A \text{ is } C \text{ or } D,$$
$$B \text{ is } cE \text{ ;}$$
$$\text{therefore, } AB \text{ is } cDE,$$

since the combination of C and cE is self-destructive.

$$(2) \quad A \text{ is } B \text{ or } C,$$
$$A \text{ is } c \text{ or } D \text{ ;}$$
$$\text{therefore, } A \text{ is } Bc \text{ or } BD \text{ or } CD.$$

$$(3) \quad X \text{ is } AB \text{ or } bce,$$
$$Y \text{ is } aBC \text{ or } DE \text{ ;}$$
$$\text{therefore, } XY \text{ is } ABDE \text{ ;}$$

for again it will be found that all the other combinations in the predicate contain contradictories.

$$(4) \quad X \text{ is } A \text{ or } Bc \text{ or } D,$$
$$Y \text{ is } aB \text{ or } Bc \text{ or } Cd \text{ ;}$$
$$\text{therefore, } XY \text{ is } Bc \text{ or } aBD \text{ or } ACd.$$

The alternatives in full are

$$AaB \text{ or } ABc \text{ or } ACd \text{ or } aBc \text{ or } Bc \text{ or } BcCd \text{ or } aBD$$
$$\text{or } BcD \text{ or } CdD.$$

But AaB, $BcCd$, CdD represent non-existent classes and may therefore be omitted. ABc, aBc, BcD are merely partial repetitions of Bc, and therefore they too may be omitted. Compare section 293.

After a very little practice, the student will find it unnecessary to write out the alternatives in full.

(5) X is A or bd or cE,
      Y is AC or aBe or d;
  therefore, XY is AC or bd or Ad or cdE.

351. The Combination of Universal Negative Propositions the subjects of which do not contain Contradictories.

No X is P,
No Y is Q,

may be taken as types of two such propositions.

By combining them we have simply,—

No XY is either P or Q.

The following are examples :

(1) No A is cd,
     No B is C or e;

       therefore, No AB is either C or e or cd.

(2) No A is bc,
     No A is Cd;

             therefore, No A is bc or Cd.

(3) No X is either aB or aC or bC or aE or bE,
     No Y is either Ad or Ae or bd or be or cd or ce;

therefore, No XY is either aB or aC or bC or aE or bE
        or Ad or Ae or bd or be or cd or ce;
   therefore, No XY is either a or b or d or e[1].

(4) No X is abd or aCd,
     No Y is bc or bD or ACD;

   therefore, No XY is abd or aCd or bc or bD or ACD.

(5) No X is aBC or aCD or aBe or aDe,
     No Y is AcD or abD or bcD or aDE or cDE;
      therefore, No XY is aBC or aD or cD or aBe[2].

       [1] Cf. section 302.   [2] Cf. section 303.
       K. L.   22

352. The Combination of Propositions the subjects of which contain Contradictories.

Such propositions cannot be directly combined in the manner just discussed. If AB is D, and bC is E, we are not really given any information by being told that what is both AB and bC is DE.

To avoid this difficulty we must by partial contraposition remove *both* the contradictories into the predicates of their respective propositions.

Thus, the propositions "AB is D" and "bC is E" may be reduced to the forms "A is b or D" and "C is B or E"; and we then have by combining them, "AC is bE or BD or DE".

Starting with such a pair of propositions as the above, it is requisite to take *both* the contradictories into the predicates, or we shall still be left with a merely identical proposition. For example, combining "AB is D" and "C is B or E", we have "ABC is B or D or E", which obviously tells us nothing.

If, however, the propositions can be reduced to such a form that the subject terms so far as they are not contradictories are the same, the predicates also being the same; then we may obtain a new proposition by just omitting the contradictory terms. Thus if we have propositions of the form AB is C, Ab is C, we may infer (since A is AB or Ab) that A is C.

The same result is also obtainable by means of the rule previously given,—

$$AB \text{ is } C, \text{ and } Ab \text{ is } C,$$
therefore, A is b or C, and A is B or C,
therefore, A is BC or bC or C,
therefore, A is C.

CHAPTER VII.

INFERENCES FROM COMBINATIONS OF COMPLEX PROPOSITIONS.

353. *Problem.*—Given any proposition, and any term X, to discriminate between the cases in which the proposition does, and those in which it does not, afford information with regard to this term.

We may assume that the original proposition is not an identical proposition.

If it is negative, let it by obversion be made affirmative.

Then, written in its most general form, it will be

Whatever is $P_1 P_2 \ldots$ or $Q_1 Q_2 \ldots$ or &c. is $A_1 A_2 \ldots$ or $B_1 B_2 \ldots$ or &c.

As shewn in section 291, this may be resolved into the independent propositions :—

$$P_1 P_2 \ldots \text{ is } A_1 A_2 \ldots \text{ or } B_1 B_2 \ldots \text{ or &c. ;}$$
$$Q_1 Q_2 \ldots \text{ is } A_1 A_2 \ldots \text{ or } B_1 B_2 \ldots \text{ or &c. ;}$$
$$\text{&c.} \qquad \text{&c.} \qquad \text{&c. ;}$$

in none of which is there any disjunction in the subject.

We may deal with these propositions separately, and if any one of them affords information with regard to X, then of course the original proposition does so.

We have then to consider a proposition of the form

$$P_1 P_2 \ldots P_n \text{ is } A_1 A_2 \ldots \text{ or } B_1 B_2 \ldots \text{ or &c.}$$

From this by contraposition we get,—

Everything is $A_1A_2 \ldots$ or $B_1B_2 \ldots$ or &c. or p_1 or $p_2 \ldots$ or p_n;

and hence, X is $A_1A_2 \ldots$ or $B_1B_2 \ldots$ or &c. or p_1 or $p_2 \ldots$ or p_n.

We may now strike out all alternatives in the predicate which contain x.

If they *all* contain x, then the information afforded us with regard to X is that it is non-existent.

If some alternatives are left, then the proposition will afford information concerning X unless, when the predicate has been simplified to the fullest possible extent, one of the alternatives is itself X uncombined with any other term, in which case it is clear that we are left with a merely identical proposition.

Now one of these alternatives will be X in any of the following cases, and only in these cases :—

First, If one of the alternatives in the predicate of the original proposition, when reduced to the affirmative form, is X.

Secondly, If any set of alternatives in the predicate of the original proposition, when reduced to the affirmative form, constitute a development of X; (since "AX or aX" is equivalent to X; "ABX or AbX or aBX or abX" is also equivalent to X; and so on).

Thirdly, If one of the alternatives in the predicate of the original proposition, when reduced to the affirmative form, contains X in combination solely with some term or terms appearing also in the subject; since in this case such alternative is equivalent to X simply.

For example,

"AB is AX or D" is equivalent to "AB is X or D."

By contraposition of this proposition in its original form
we have,—

Everything is AX or D or a or b,

but, (cf. section 293), "AX or a" is equivalent to "X or a."

Fourthly, If one of the terms originally contained in the
subject is x; since in that case we should after contra-
position have X as one of the alternatives in the predicate.

The above may now be summed up in the propo-
sition :—

*Any non-identical proposition will afford information with
regard to any term X, unless (after it has been brought to the
affirmative form), (1) one of the alternatives in the predicate is
X, or (2) any set of alternatives in the predicate constitute a
development of X, or (3) any alternative in the predicate con-
tains X in combination with such terms only as appear also in
every alternative in the subject, or (4) every alternative in the
subject contains x.*

If, after the proposition has been reduced to the affirma-
tive form, the simplifications noticed in section 293 have
been effected, then the criterion becomes more simply,—

*Any non-identical proposition will afford information with
regard to any term X, unless, (after it has been brought to the
affirmative form, and its predicate so simplified that it contains
no superfluous terms), one of the alternatives in the predicate is
X, or every alternative in the subject contains x.*

If instead of X we have a complex term XYZ, then no
part of this term must appear as an alternative in the predi-
cate, and there must be at least one alternative in the
subject which does not contain the contradictory of any
part of this complex term : *i.e.*, no alternative in the predi-
cate must be X, Y, or Z, and some alternative in the subject
must contain neither x, y, nor z.

The above criterion is of simple application.

354. Say, by inspection, which of the following propositions give information concerning A, aB, b, bCd, respectively :—

Ab is bCd or c ;

bd is A or bC or abc ;

Whatever is a or B is c or D ;

Whatever is Ab or bc is bE or cE or e ;

X is AX or ab or Bc or Cd.

355. *Problem.*—Given any number of propositions involving any number of terms, to determine what is all the information that they jointly afford with regard to any given term or combination of terms that they contain.

The great majority of direct problems[1] involving complex propositions may be brought under the above general form. If the student will turn to Boole, Jevons, or Venn, he will find that it is by them treated as the central problem of Symbolic Logic.

A general method of solution is as follows:—

Let X be the term concerning which information is desired. Find what information each proposition gives separately with regard to X, thus obtaining a new set of propositions of the form

$$X \text{ is } P_1 \text{ or } P_2 \text{ or } \ldots \text{ or } P_n.$$

This is always possible by the aid of the rules given in the preceding chapters[2]. It should be remembered that in section 353 we have discriminated the cases in which any given proposition really affords information with regard to X.

[1] Inverse problems are discussed in chapter XII.

[2] The importance of these rules, especially of those relating to Contraposition, is now made more apparent.

Those propositions which do not do so may of course be altogether left out of account.

Next combine the propositions thus obtained in the manner indicated in section 350. This will give the desired solution.

The method might be varied by bringing the propositions to the form,—

No X is Q_1 or Q_2 or ... or Q_n,

then combining as in section 351, and finally obverting the result. It will depend on the form of the original propositions whether this variation is desirable[1].

If information is desired with regard to several terms, it may be found convenient to bring all the propositions to the form,—

Everything is P_1 or P_2 ... or P_n;

and to combine them at once, getting in a single proposition a summation of all the information given by the separate propositions taken together. From this we may immediately obtain all that is known concerning X by leaving out every alternative that contains x, all that is known concerning Y by leaving out every alternative that contains y, and so on.

The following may be taken as a simple example of the method. It is adapted from Boole (*Laws of Thought*, p. 118).

"Given 1st, that wherever the properties A and B are combined, either the property C, or the property D, is present also; but they are not jointly present: 2nd, that wherever the properties B and C are combined, the properties A and D are either both present with them, or both

---

[1] The *second* method bears a somewhat close resemblance to Jevons's Indirect Method; though it is not quite the same. The *first* method however is quite distinct from Jevons's method.

absent. Shew that where A is present, either B or C is absent."

The premisses may be written,—

$$AB \text{ is } Cd \text{ or } cD; (1)$$
$$BC \text{ is } AD \text{ or } ad. (2)$$

Then, we may immediately obtain,—

from (1), A is b or Cd or cD;

and from (2), A is b or c or D^1;

therefore (by combining these), A is b or cD;

therefore, A is b or c;

which is the desired result.

This is a simple example; but many more complicated ones will be found in the following pages.

The method here described will I think in nearly every case be found less laborious than that employed by Jevons[2], —namely, the writing down all the possible *a priori* alternatives so far as the terms involved are concerned, and then striking out those that are inconsistent with the premisses. Also, while it neither requires that propositions shall be reduced to the form of equations, nor involves the use of mathematical symbols or diagrams, I have not in practice found it less effective than the methods of Boole and Venn[3].

I shall further endeavour to shew in subsequent sections, how special results may frequently be obtained in a still simpler way by the aid of various formal processes. In some of the examples that follow both the general method and special methods are employed.

---

[1] An intermediate step might be introduced here, namely, ABC is D.

[2] *Pure Logic*, pp. 44, 45; *Principles of Science*, chapter VI.

[3] At the same time of course these methods have a peculiar interest and significance of their own.

While the special methods are as a rule to be preferred when they have been discovered, it generally takes some time and ingenuity to discover them. On the other hand, the general method above described may be always immediately applied without any preliminary study of the case. Also, while special methods are useful to establish given results, we can ordinarily be satisfied that we have a *complete* solution with regard to any term only when we have employed the general method.

CHAPTER VIII.

PROBLEMS INVOLVING THREE TERMS.

356. Given that everything is either Q or R, and that all R is Q, unless it is not P, prove that all P is Q.

The premises may be written,—

$$r \text{ is } Q, (1)$$
$$PR \text{ is } Q. (2)$$
$$\text{By } (1), Pr \text{ is } Q,$$
$$\text{by } (2), PR \text{ is } Q;$$
$$\text{but } P \text{ is } Pr \text{ or } PR;$$
$$\text{therefore, } P \text{ is } Q.$$

357. Where A is present, B and C are either both present at once or absent at once; and where C is present, A is present. Describe the class not-B under these conditions. [Jevons, *Studies*, p. 204.]

The premises are,—

$$A \text{ is } BC \text{ or } bc, (1)$$
$$C \text{ is } A. (2)$$
$$\text{By } (1), Ab \text{ is } c,$$
$$\text{by } (2), ab \text{ is } c,$$
$$\text{therefore, } b \text{ is } c.$$

The solution is, therefore, "Where B is absent, C also will be absent."

358. Given (1) P is QR, (2) p is qr; shew that (3) Q is RP, (4) R is PQ.

359. Given (1) R is P or pq, (2) q is R or Pr, (3) qR is P; shew that p is Qr.

360. Whenever X is present, Y and Z are both present; and whenever X is absent, Y and Z are both absent. What can thence be inferred with regard to the relation between Y and Z?

361. (1) Wherever there is smoke there is also fire or light;

(2) Wherever there is light and smoke there is also fire;

(3) There is no fire without either smoke or light.

Given the truth of the above propositions, what is all that you can infer with regard to (i) circumstances where there is smoke; (ii) circumstances where there is not smoke; (iii) circumstances where there is not light?   [w.]

Let A = circumstances where there is smoke,

$\quad B$ = circumstances where there is light,

$\quad C$ = circumstances where there is fire.

The premisses are,—

$$A \text{ is } B \text{ or } C, \quad (1)$$
$$AB \text{ is } C, \quad (2)$$
$$C \text{ is } A \text{ or } B. \quad (3)$$

(1) and (2) yield A is C.

By (3) ab is c; therefore, a is B or c.

By (1) and (3), b is a or C, and also A or c; therefore, b is AC or ac.

We have then,—

(i) Where there is smoke, there is fire;

(ii) Where there is not smoke, there is either light or there is no fire;

(iii) Where there is no light, there is either both fire and smoke or neither fire nor smoke.

362. Shew the equivalence between the two sets of propositions,—

(i) A is BC,
     B is AC,
     C is AB.

(ii) A is BC,
      a is bc.

B is AC, C is AB, give by contraposition a is bc.

a is bc gives by contraposition B is A, C is A; and since A is BC, we have B is AC, C is AB.

363. Shew the equivalence between the following sets of propositions:—

(1) b is aC,
     c is aB;

(2) A is BC,
     b is aC;

(3) A is BC,
     c is aB.

364. Shew the equivalence between the following sets of propositions :—

(1) a is BC,
     b is AC,
     C is Ab or aB;

(2) a is BC,
       B is Ac or aC,
       c is AB;

(3) A is Bc or bC,
       b is AC,
       c is AB.

365. A is Bc or bC, b is AC, c is AB. Shew that all the information given by the combination of these propositions is also given by the propositions,—A is b or c, b is A, c is AB or ab.

366. Apply the method of solution described in section 355 to ordinary syllogisms in *Barbara, Cesare, Camenes*.

Barbara has for its premisses,—

$$(1)\ M \text{ is } P,$$
$$(2)\ S \text{ is } M.$$
By (1), S is m or P;
by (2), S is M;
therefore, S is MP;
therefore, S is P.

Cesare,   (1) No P is M,
$$(2)\ S \text{ is } M.$$
By (1), S is m or p;
by (2), S is M;
therefore, S is Mp;
therefore, S is not P.

Camenes,   (1) P is M,
$$(2)\ \text{No } M \text{ is } S.$$
By (1), S is M or p;
by (2), S is m;
therefore, S is mp;
therefore, No S is P.

367. Assign propositions concerning the terms *gem, rich, rare,* whose aggregate force shall be such that no further assertion can be made about the same terms without contradicting the propositions assigned.

[L.]

Jevons regards any two or more propositions as inconsistent when they involve the total disappearance of any term, positive or negative, (*Studies in Deductive Logic,* p. 181); and in dealing with such a question as the above, it is perhaps a convenient criterion to take. It is equivalent in this instance to the assumption that there exist gems and not gems, rich things and not rich things, rare things and not rare things. It is an assumption, however, and as such should always be explicitly stated, when made. Compare Part II. Chapter VIII. The reason why it is convenient to make it here is that, (as shewn in section 106), on the supposition that All S is P does not itself imply the existence of S, All S is P and No S is P are either inconsistent, or between them deny the existence of S. We must therefore exclude the latter possibility, if we wish to be able to say definitely that two such propositions are inconsistent.

The given question is now solved by the *three* propositions :—

(1)   All gems are rich and rare ;
(2)   All rich things are rare gems ;
(3)   All rare things are rich gems.

We can make no further assertion regarding these terms or their negatives which are not either implied by the above, or else inconsistent with them ; since the only classes which they allow to remain are rich rare gems and not rich not rare not gems.

The *two* propositions,—

(i)   All gems are rich and rare ;

(ii) All things not gems are neither rich nor rare;
also afford a solution.

The equivalence between (1), (2), (3), and (i), (ii), has been already shewn in section 362.

The student should now find other sets of propositions, not equivalent to the above, which also afford a solution of the given problem.

CHAPTER IX.

368. Suppose that an analysis of the properties of a particular class of substances has led to the following general conclusions, viz.:

1st, That wherever the properties A and B are combined, either the property C, or the property D, is present also; but they are not jointly present.

2nd, That wherever the properties B and C are combined, the properties A and D are either both present with them, or both absent.

3rd, That wherever the properties A and B are both absent, the properties C and D are both absent also; and *vice versâ*, where the properties C and D are both absent, A and B are both absent also.

Let it then be required from the above to determine what may be concluded in any particular instance from the presence of the property A with respect to the presence or absence of the properties B and C, paying no regard to the property D.

[Boole, *Laws of Thought*, pp. 118—120; compare also Venn, *Symbolic Logic*, pp. 276—278.]

One solution has already been given in section 355. We might also proceed as follows. The premisses are:

$$AB \text{ is } Cd \text{ or } cD, \qquad \text{(i)}$$
$$BC \text{ is } AD \text{ or } ad, \qquad \text{(ii)}$$
$$ab \text{ is } cd, \qquad \text{(iii)}$$
$$cd \text{ is } ab. \qquad \text{(iv)}$$

By (i), No AB is CD,
therefore, No A is BCD. $\qquad$ (1)

By (ii), No BC is Ad,
therefore, No A is BCd. $\qquad$ (2)

Combining (1) and (2), we have,—

No A is BC,

i.e., All A is b or c.

This solves the problem as set.

Boole also shews that All bC is A. This is a contrapositive of (iii). We have not required to make use of (iv) at all.

369. Given the same premisses as in the preceding section, prove that :—

(1) Wherever the property C is found, either the property A or the property B will be found with it, but not both of them together.

(2) If the property B is absent, either A and C will be jointly present, or C will be absent.

(3) If A and C are jointly present, B will be absent.   [Boole, *Laws of Thought*, p. 129.]

First, $\qquad$ By (i), ABC is d,
by (ii), ABC is D ;
i.e., there is no such thing as ABC;
i.e., C is a or b.

K. L. $\hfill$ 23

Also, by contraposition of (iii), C is A or B;
therefore, C is Ab or aB. (1)

Secondly,   By (iii), b is A or c,
therefore, b is AC or c. (2)

Thirdly, We have shewn that it follows from (i) and (ii) that there is no such thing as ABC,
therefore, AC is b. (3)

370. It is known of certain things that (1) where the quality A is, B is not; (2) where B is, and only where B is, C and D are. Derive from these conditions a description of the class of things in which A is not present, but C is. [Jevons, *Studies*, p. 200.]

The premises are,—

(1) A is b;
(2) B is CD;
(3) CD is B.

(1) affords no information with regard to aC. Cf. section 353.

But by (2), aC is b or D;
and by (3), aC is B or d;
therefore, aC is BD or bd.

371. Taking the same premises as in the previous section, draw descriptions of the classes Ac, ab, and cD.   [Jevons, *Studies*, p. 244.]

We have   (1) A is b;
(2) B is CD;
(3) CD is B.
By (1), Ac is b;
by (2), Ac is b;
(3) affords no information with regard to Ac.

> By (3), *ab* is *c* or *d*.
> By (1), *cD* is *a* or *b*;
> by (2), *cD* is *b*;
> therefore, *cD* is *b*.

We can obtain no further information with regard to *ab* and *cD*.

The desired results, therefore, are,—

> *Ac* is *b*;
> *ab* is *c* or *d*;
> *cD* is *b*.

372. There is a certain class of things from which *A* picks out the '*X* that is *Z*, and the *Y* that is not *Z*,' and *B* picks out from the remainder 'the *Z* which is *Y* and the *X* that is not *Y*.' It is then found that nothing is left but the class '*Z* which is not *X*.' The whole of this class is however left. What can be determined about the class originally?

[Venn, *Symbolic Logic*, pp. 267, 8.]

The chief difficulty in this problem consists in the accurate statement of the premisses. Call the original class *W*. We then have,—

$$W \text{ is } XZ \text{ or } Yz \text{ or } YZ \text{ or } Xy \text{ or } xZ, \quad (1)$$
$$xZ \text{ is } W. \quad (2)$$

No *xZ* is *WXZ* or *WYz* or *WYZ* or *WXy*,

i.e., (leaving out such part of this statement as is merely identical),

$$\text{No } xZ \text{ is } WYZ. \quad (3)$$

We may now proceed as follows :—

By (3),   No *W* is *xYZ*;   (4)

By (1),   No *W* is *xyz*.   (5)

Combining this with (4), we find that the class did not originally contain any not-X that was either both Y and Z or neither Y nor Z.

(2) affords no information regarding the class W, except that everything that is Z but not X is contained within it. The student may however notice that from this proposition in conjunction with (3), it may be deduced that all YZ is X.

373. At a certain town where an examination is held, it is known that,

(1) Every candidate is either a junior who does not take Latin, or a Senior who takes Composition.

(2) Every junior candidate takes either Latin or Composition.

(3) All candidates who take Composition, also take Latin, and are juniors.

Shew that if this be so there can be no candidates there.   [Venn, *Symbolic Logic*, pp. 270, 1.]

Let X = candidate,

A = junior, so that a = senior,

B = taking Latin,

C = taking Composition.

We then have,—

$$X \text{ is } Ab \text{ or } aC; \qquad (1)$$
$$XA \text{ is } B \text{ or } C; \qquad (2)$$
$$XC \text{ is } AB. \qquad (3)$$

(2) and (3) give XA is B;

therefore, No X is Ab;

also by (3),   No X is aC.

It therefore follows from (1) that there can be no such thing as X.

374.   Given (1) x is yz; (2) ZW is y; (3) y is ZW; (4) z is W; (5) XW is YZ; shew that (i) Nothing is W, (ii) Everything is XYZ.

[Venn, *Symbolic Logic*, pp. 271, 2.]

By (5), XW is YZ;
but by (2), No W is YZ;
therefore, Nothing is XW.

By (1), xW is yz;
but by (3), Nothing is yz;
therefore, Nothing is xW.

But W is XW or xW;
therefore, Nothing is W.   (i)

By (1), x is yz;
but by (3), Nothing is yz;
therefore, Nothing is x.

By (3), y is W,
and by (4), z is W;
but by (i), Nothing is W;
therefore, Nothing is y or z;
therefore, Nothing is x, y or z;
i.e., Everything is XYZ.   (ii)

375.   If thriftlessness and poverty are inseparable, and virtue and misery are incompatible, and if thrift be a virtue, can any relation be proved to exist between misery and poverty? If moreover all thriftless people are either virtuous or not miserable, what follows?   [V.]

Let A = thriftless,
B = poor,
C = virtuous,
D = miserable.

Then the premisses as first given may be written,—

$$A \text{ is } B^1, \quad (1)$$
$$B \text{ is } A^1, \quad (2)$$
$$\text{No } C \text{ is } D, \quad (3)$$
$$a \text{ is } C. \quad (4)$$

Can we now find any relation between B and D?

We may proceed by finding *all* that we can assert concerning B and D respectively.

$$\text{By (2), } B \text{ is } A ;$$
$$\text{by (3), } B \text{ is } c \text{ or } d^2 ;$$
$$\text{therefore, } B \text{ is } Ac \text{ or } Ad ;$$
$$\text{therefore, If } B \text{ is } D, \text{ it is } Ac ;$$

i.e., If poverty is accompanied by misery, it is also accompanied by thriftlessness, and it is not accompanied by virtue.

Again,   by (1), D is a or B ;
$$\text{by (2), } D \text{ is } A \text{ or } b ;$$
$$\text{therefore, } D \text{ is } AB \text{ or } ab ;$$
$$\text{by (3), } D \text{ is } c ;$$
$$\text{therefore, } D \text{ is } ABc \text{ or } abc ;$$
$$\text{by (4), } D \text{ is } A \text{ or } C ;$$
$$\text{therefore, } D \text{ is } ABc ;$$

i.e., Misery is always accompanied by poverty ; or, misery is never found unaccompanied by poverty.

This result might also be obtained by two ordinary syllogisms in *Barbara*, as follows:

---

[1] If we adopted the equational rendering of propositions, which however I have intentionally avoided, "A is B" and "B is A" would of course be summed up in "$A = B$." In cases of this kind, the equational rendering is at its best.

[2] (1) gives no information regarding B; and, so far as B is concerned, (4) merely repeats part of the information given by (2).

> By (3), D is c;
> by (4), c is A;
> therefore, D is A;
> by (1), A is B;
> therefore, D is B.

If to the given premisses we now add (5) A is C or d,
we find that D is both A and a, a result which must be
interpreted as affirming the non-existence of D; *i.e.*, There
is no such thing as misery.

It will be a more complete answer to the latter part of
the question to note the full result of combining all the given
premisses.

> By (1), Everything is a or B;
> by (2), Everything is A or b;
> therefore, Everything is AB or ab;
> by (3), Everything is c or d;
> therefore, Everything is ABc or ABd or abc or abd;
> by (4), Everything is A or C;
> therefore, Everything is ABc or ABd or $abCd$;
> by (5), Everything is a or C or d;
> therefore, Everything is ABd or $abCd$.

This gives us :—

> A is Bd;
> a is bCd;
> B is Ad;
> b is aCd;
> C is ABd or abd;
> c is ABd;
> There is no such thing as D;
> d is AB or abC.

360 COMPLEX INFERENCES. [PART IV.

376. A given class is made up of those who are
not male guardians, nor female ratepayers, nor lodgers
who are neither guardians nor ratepayers. How can
we simplify the description of this class if we know
that all guardians are ratepayers, that every person
who is not a lodger is either a guardian or a rate-
payer, and that all male ratepayers are guardians?

[v.]

Let $\quad\quad\quad\quad X=$ the given class,
$\quad\quad\quad\quad\quad\quad A=$ male,
$\quad\quad\quad\quad\quad\quad B=$ guardian,
$\quad\quad\quad\quad\quad\quad C=$ ratepayer,
$\quad\quad\quad\quad\quad\quad D=$ lodger.

Then X is made of those who are

$\quad\quad\quad\quad$ not AB nor aC nor bcD ;

that is, X is made up of those who are

$\quad\quad\quad\quad aBc$ or AbC or acd or Abd or bcd.

But we are told that,—

$\quad\quad\quad\quad$ (1) $\quad B$ is C ;
$\quad\quad\quad\quad$ (2) $\quad d$ is B or C ;
$\quad\quad\quad\quad$ (3) $\quad AC$ is B.

From (1), it follows that there is no aBc; from (2), that
there is no bcd ; from (3), that there is no AbC; from (2)
and (3) taken together that there is no Abd ; from (1) and
(2) taken together that there is no acd.

It therefore follows that the given class is itself non-
existent.

We might arrive at the same result as follows :—

$\quad\quad\quad\quad$ By (1), Everything is b or C ;
$\quad\quad\quad$ by (2), Everything is B or C or D ;
$\quad\quad\quad$ therefore, Everything is bD or C ;

by (3), Everything is a or B or c ;

therefore, Everything is abD or bcD or aC or BC ;

but abD is aC or bcD[1],

and BC is AB or aC[2] ;

therefore, Everything is aC or bcD or AB ;

which again shews that the given class is non-existent.

377. Given that everything that is Q but not S is either both P and R or neither P nor R and that neither R nor S is both P and Q, shew that no P is Q.

378. Where C is present, A, B and D are all present ; where D is present, A, B and C are either all three present or all three absent. Shew that when either A or B is present, C and D are either both present or both absent. How much of the given information is superfluous so far as the desired conclusion is concerned ?

379. Every voter is both a ratepayer and occupier, or not a ratepayer at all.

If any voter who pays rates is an occupier, then he is on the list.

No voter on the list is both a ratepayer and an occupier.

Examine the results of combining these three statements.   [v.]

380. At a certain examination, all the candidates who were entered for Latin were also entered for either

---

[1] Since, by the law of Excluded Middle, abD is $abCD$ or $abcD$.

[2] Since, by the law of Excluded Middle, BC is ABC or aBC.

Greek, French, or German, but not for more than one
of these languages ; all the candidates who were not
entered for German were entered for two at least of
the other languages ; no candidate who was entered
for both Greek and French was entered for German,
but all candidates who were entered for neither
Greek nor French were entered for Latin. Shew
that all the candidates were entered for two of the
four languages but none for more than two.

381. AB is D, ab is cd, c is ABD or abd, D is
AB. All the information given by these propositions
is also given by the propositions,—ABC is D, abd is
c, c is AD or abd, D is AB or ac or Bc ; and *vice
versâ*.

382. Shew that the following sets of propositions
are equivalent :—

(1) a is b or c ; b is aCd ; c is aB ; D is c.
(2) A is BC ; b is aC ; C is ABd or abd.
(3) A is B ; B is A or c ; c is aB ; D is c.
(4) b is aC ; A is C ; C is d ; aC is b.
(5) c is aB ; D is aB ; A is B ; aB is c.
(6) A is BC ; BC is A ; D is Bc ; b is C.

383. Let the observation of a class of natural productions be supposed to have led to the following general results.

1st. That in whichsoever of these productions the properties A and C are missing, the property E is found, together with one of the properties B and D, but not with both.

2nd. That wherever the properties A and D are found while E is missing, the properties B and C will either both be found, or both be missing.

3rd. That wherever the property A is found in conjunction with either B or E, or both of them, there either the property C or the property D will be found, but not both of them. And conversely, wherever the property C or D is found singly, there the property A will be found in conjunction with either B or E, or both of them.

Shew that it follows that *In whatever substances the property A is found, there will also be found either the property C or the property D, but not both, or else the*

properties B, C, and D, will all be wanting. And conversely, *where either the property C or the property D is found singly, or the properties B, C, and D, are together missing, there the property A will be found.*

[Boole, *Laws of Thought*, pp. 146—148. Cp. also
Venn, *Symbolic Logic*, pp. 280, 281.]

The premisses are as follows:—

<div style="text-align:center">

1st, All ac is BdE or bDE;   (i)
2nd, All ADe is BC or bc;   (ii)
3rd, All AB is Cd or cD;   (iii)
All AE is Cd or cD;   (iv)
All Cd is AB or AE;   (v)
All cD is AB or AE.   (vi)

</div>

We are required to prove:—

<div style="text-align:center">

All A is Cd or cD or bcd;   (α)
All Cd is A;   (β)
All cD is A;   (γ)
All bcd is A.   (δ)

</div>

First, By (iii) and (iv), If A is B or E it is Cd or cD;
therefore, A is Cd or cD or be.   (1)

By (ii), Ae is either BC or bc or d;
therefore, Abe is bc or d;
therefore, Abe is bce or bde.   (2)

By (v), Cd is B or E;
therefore, C is B or D or E;
therefore (by contraposition), bde is c;
therefore, bde is bcd;
therefore, If Abe is bde it is bcd.   (3)

Again by (vi), cD is B or E;
therefore (as above), bce is d;
therefore, If Abe is bce it is bcd.   (4)

Therefore, by (2), (3), and (4), *Abe* is *bcd*;

therefore from (1), *A* is either *Cd* or *cD* or *bcd*.   (α)

Secondly, (β) and (γ) follow immediately from (v) and (vi).

Thirdly,   from (i), we have directly, No *ac* is *bd*;

therefore (by conversion), No *bcd* is *a*;

therefore, All *bcd* is *A*.   (δ)

The first of the desired results might also be obtained as follows:—

As before we may shew that

$$A \text{ is } Cd \text{ or } cD \text{ or } be;$$

and we therefore have what is required if we can shew that

Abe is *cd*.

By (ii), *Abe* is *c* or *d*;

by (v), *Abe* is *c* or *D*;

therefore, *Abe* is *c*;

by (vi), *Abe* is *C* or *d*;

therefore, *Abe* is *cd*.

We have here employed a modification of the general method described in section 355.

We might also by this method obtain a *complete* solution of the problem so far as *A* is concerned.

(i) gives no information whatever with regard to *A*[1].

But by (ii), *A* is *BC* or *bc* or *d* or *E*;

by (iii), *A* is *b* or *Cd* or *cD*;

therefore, *A* is *Cd* or *bc* or *bd* or *bE* or *cDE*;

by (iv), *A* is *Cd* or *cD* or *e*;

therefore, *A* is *Cd* or *cDE* or *bcD* or *bce* or *bde*;

by (v), *A* is *B* or *E* or *c* or *D*;

therefore, *A* is *cDE* or *bcD* or *bce* or *BCd* or *CdE*;

---

[1] Since *a* appears in the subject.   Cf. section 353.

by (vi), *A* is *B* or *E* or *C* or *d*;

therefore, *A* is *cDE* or *bcde* or *BCd* or *CdE*[1].

This includes the partial solution with regard to *A*,—

A is *Cd* or *cD* or *bcd*.

Boole contents himself with this because he has started with the intention of eliminating *E* from his conclusion.

384. Given the same premisses as in the preceding section, prove that *If the property A is absent and C present, D is present.*

[Boole, *Laws of Thought*, p. 148.]

By (v), *Cd* is *A*;

therefore (by contraposition), *aC* is *D*.

385. Given the same premisses as in section 383, shew that,—

1st. *If the property B be present in one of the productions, either the properties A, C, and D, are all absent, or some one alone of them is absent.* And conversely, *if they are all absent it may be concluded that the property B is present.*

2nd. *If A and C are both present or both absent, D will be absent, quite independently of the presence or absence of B.*   [Boole, *Laws of Thought*, p. 149.]

---

[1] To shew that this method is not very laborious, it may perhaps be worth mentioning that no step in my original working is omitted in the above. In the first instance, without any knowledge of the solution that would result, I obtained it by aid of the steps here inserted without erasure or rough working of any kind. I am doubtful whether by any other method the result could be reached more expeditiously. It is probable that at first the student may require to insert some other steps in the reasoning, and that the possible simplifications may not immediately occur to him. But this will be remedied by a very little practice.

We have to shew,—

      (α) B is acd or aCD or AcD or ACd;

      (β) acd is B;

      (γ) AC is d;

      (δ) ac is d.

First,   By (iii), B is Cd or cD or a;

therefore, B is ACd or AcD or a.   (1)

      By (i), ac is Bd or bD;

      therefore, No ac is BD;

      therefore, No aB is cD.   (2)

         By (v), Cd is A;

therefore (by contraposition), a is c or D;

        therefore, No a is Cd;

        therefore, No aB is Cd.   (3)

      By (2) and (3), No aB is cD or Cd;

      therefore, All aB is cd or CD.

Combining this result with (1), we have,—

      B is ACd or AcD or acd or aCD.   (α)

Secondly, From (i) we have directly, acd is BdE;

            therefore, acd is B.   (β)

Thirdly,   By (ii), No ADe is bC;

      therefore, No AC is bDe.   (1)

      By (iii), No AB is CD;

      therefore, No AC is BD.   (2)

      By (iv) No AE is CD;

      therefore, No AC is DE.   (3)

Therefore, by (1), (2), and (3), If AC is D it is neither be nor B or E; but (by the law of excluded middle),

      All AC is either B or E or be;

        therefore, No AC is D;

        therefore, All AC is d.   (γ)

Lastly,   By (vi), cD is A ;

                    therefore (by contraposition), ac is d.   (δ)

For complete solutions with regard to B, *acd, AC, ac,* see the following section.

386. Given the same premisses as in section 383, obtain complete solutions with regard to B, *acd, AC, ac.*

Complete solutions with regard to B, *acd, AC, ac,* may be obtained by the general method described in section 355 as follows :—

First,   By (i), B is dE or A or C ;

                    by (ii), B is C or a or d or E ;

            therefore, B is C or dE or Ad or AE ;

                    by (iii), B is Cd or cD or a ;

        therefore, B is Cd or aC or adE or $AcDE$;

(iv) gives no information with regard to B that is not already given by (iii) ;

                    by (v), B is A or c or D ;

        therefore, B is $AcDE$ or ACd or aCD or $acdE$.

(vi) gives no further information with regard to B.

The above includes the special solution given by Boole,—

            B is *acd* or aCD or AcD or ACd.

Secondly, By (i) *acd* is BE.

None of the other propositions give any information with regard to *acd*[1]. This then is the complete solution so far as *acd* is concerned.

---

[1] Since the subjects of all these propositions contain either A, C, or D. Cf. section 353.

Thirdly,   By (ii), AC is B or d or E;
      by (iii), AC is b or d;
    therefore, AC is d or bE;
      by (iv), AC is d or e;
      therefore, AC is d;
    by (v), AC is B or E or D;
    therefore, AC is Bd or dE.

Lastly,   By (i), ac is BdE or bDE;
      by (vi), ac is d;
    therefore, ac is BdE.

387.   Every A is one only of the two B or C, D is both B and C except when B is E and then it is neither; therefore no A is D.   [De Morgan.]

This example, originally given by De Morgan, (using however different letters), and taken by Professor Jevons to illustrate his symbolical method, (*Principles of Science*, Vol. 1, p. 117; *Studies in Deductive Logic*, p. 203), is chosen by Professor Croom Robertson to shew that "the most complex problems can, as special logical questions, be more easily and shortly dealt with upon the principles and with the recognised methods of the traditional logic" than by Jevons's system.

"The mention of E as E has no bearing on the special conclusion A is not D and may be dropt, while the implication is kept in view; otherwise, for simplification, let BC stand for 'both B and C,' and bc for 'neither B nor C.' The premisses then are,—

      (1) D is either BC or bc,
      (2) A is neither BC nor bc,

which is a well-recognised form of Dilemma with the conclusion A is not D. Or, by expressing (2) as A is not

K. L.   24

either BC or bc, the conclusion may be got in Camestres. If it be objected that we have here by the traditional processes got only a special conclusion, it is a sufficient reply that any conclusion by itself must be special. What other conclusion from these premisses is the common logic powerless to obtain?" (*Mind*, 1876, p. 222.)

The solution is also obtainable as follows,—

By the first premiss, A is Bc or bC, and by the second, A is BC or bc or d;

<div align="center">

therefore, A is Bcd or bCd,

therefore, A is d.

</div>

Professor Robertson's solution is in this case preferable. But I append the above as a further illustration of my own method. Compared with the problem of Boole's just discussed or with the problems that follow, this of De Morgan's is not particularly complex.

388. Suppose it known that,—

(1) Where B is present and C absent, either D is present or E is absent;

(2) Where A and D are present and C absent, B is present;

(3) Where B is absent and C present, A is present;

(4) Where C and D are present, A is absent or B is present;

(5) Where E is present and D absent, A and C are not both present nor are B and C both absent;

(6) Where B is present and D absent, C is absent;

(7) Where A is present and E absent, either B or D is present;

then we can shew that,—

(i)   Where A is present, B is present;

(ii)   Where B is absent, C is absent;

(iii)   Where C is present, D is present;

(iv)   Where D is absent, E is absent.

First,   By (2), AcD is B;

by (4), CD is a or B;

therefore, ACD is B;

therefore, AD is B.

Again, by (5), No dE is AC;

therefore, AdE is c;

therefore, AdE is cdE.

But also by (5), cdE is B;

therefore, AdE is B;

by (7), Ade is B;

therefore, Ad is B.

But we have shewn above that AD is B;

therefore, A is B.   (i)

Secondly,   By (3), bC is A;

therefore, ab is c.

By (i), b is a;

therefore, b is ab;

therefore, b is c.   (ii)

Thirdly,   By (6), Bd is c;

therefore, BC is D.

By (ii), C is B;

therefore, C is BC;

therefore, C is D.   (iii)

Fourthly,   By (1), Bc is D or e;

therefore, Bcd is e.

24—2

By (5), No dE is bc;
   therefore, bcd is e;
   therefore, cd is e.

   By (iii), d is cd;
   therefore, d is e.   (iv)

389.   Given that,—

(1) Where A and C are absent, D and E are absent;

(2) Where B and D are present, C is present or else A is present and E absent;

(3) Where B is absent, A and C are present or A and D are absent;

(4) Where C and E are present, A, B, and D are absent;

(5) Where D is absent and E is present, A is absent;

(6) Where C and E are absent, either A or D is absent.

Prove that,—

(i) Where A is present and B absent, C is present;

(ii) Where B is absent and D present, A and C are present and E is absent;

(iii) Where C is absent, D and E are absent;

(iv) Where E is present, A, B, and D are absent and C is present.

First,   By (3), b is AC or ad;
               therefore, Ab is C.   (i)

Secondly,   By (3), b is AC or ad;
               therefore, bD is AC.

By (4), CE is abd;

therefore, Everything is c or e or abd;

therefore, bD is c or e;

but as shewn above, bD is AC;

therefore, bD is ACe.   (ii)

Thirdly,   By (1), ac is de;

therefore, c is A or de.

By (2). BD is C or Ae;

therefore, Everything is b or d or C or Ae;

therefore, c is b or d or Ae;

therefore, c is Ae or Ab or Ad or de.

By (3), b is AC or ad;

therefore, c is B or ad;

therefore, c is ABe or ABd or de.

By (5), dE is a;

therefore, c is D or e or a;

therefore, c is ABe or de;

by (6), c is a or d or E;

therefore, c is de.   (iii)

Fourthly,   By (1), E is A or C;

By (2), E is b or d or C;

therefore, E is Ab or Ad or C;

by (3), E is B or AC or ad;

therefore, E is AC or ABd or BC or aCd;

by (4), E is abd or c;

therefore, E is $ABcd$ or $abCd$;

by (5), E is a or D;

therefore, E is $abCd$.   (iv)

390.   Given that,—

$$ad \text{ is } b \text{ or } e;$$
$$b \text{ is } AcD \text{ or } ad \text{ or } cde;$$
$$c \text{ is } AbD \text{ or } bde;$$
$$e \text{ is } aBCd \text{ or } bcd;$$

shew that, A is BCE or $bcDE$ or $bcde$;

$$a \text{ is } BCDE \text{ or } BCde \text{ or } bCdE \text{ or } bcde;$$
$$B \text{ is } ACE \text{ or } aCde \text{ or } CDE;$$
$$C \text{ is } ABE \text{ or } aBde \text{ or } abdE \text{ or } BDE.$$

391.   (i)   Where P is present while Q and R are absent, S and T are present.

(ii) Q and R are always present or absent together.

(iii)   Where Q, R and S are all present, P and T are either both present or both absent.

(iv)   Where Q and R are present while S is absent, P is present and T is absent.

(v)   Where S is present while P, Q and R are all absent, T is present.

Then,—

(1)   Where P is present while Q is absent, S and T are present and R is absent.

(2)   Where P is present while S is absent, Q and R are present and T is absent.

(3)   Where P is absent while Q is present, R and S are present and T is absent.

(4)   Where P is absent while T is present, Q and R are both absent.

(5)   Where Q is present while S is absent, P and R are present and T is absent.

(6)   Where Q and T are both present, P, R and S are also present.

(7)   Where Q is absent while S is present, R is absent and T present.

(8)   Where Q and S are both absent, P and R are both absent.

(9)   Where Q and T are both absent, P, R and S are also absent.

(10)   Where S is absent while T is present, P, Q and R are all absent.

By (i), Everything is p or Q or R or ST;

By (ii), Everything is QR or qr;
therefore, Everything is QR or pqr or $qrST$;

By (iii), Everything is q or r or s or PT or pt;
therefore, Everything is pqr or $qrST$ or QRs or $PQRT$ or $pQRt$;

By (iv), Everything is q or r or S or Pt;
therefore, Everything is pqr or $qrST$ or $PQRst$ or $PQRST$ or $pQRSt$;

By (v), Everything is s or P or Q or R or T;
therefore, Everything is $pqrs$ or $pqrT$ or $qrST$ or $PQRst$ or $PQRST$ or $pQRSt$.

The desired results follow from this immediately.

392.   Given that,—

$$A \text{ is } Bc \text{ or } bC;$$
$$B \text{ is } DE \text{ or } de;$$
$$C \text{ is } De;$$

shew that,—

> A is $BcDE$ or $Bcde$ or $bCDe$;
> BcD is E;
> abd is c;
> cd is Be or ab;
> bCD is e.

[Jevons, *Pure Logic*, p. 66.]

393. At a certain examination it was observed that,—

(i) all candidates who were entered for Greek were entered also for Latin;

(ii) all candidates who were not entered for Greek were entered for English and French, and if they were also entered for Latin, they were entered for German;

(iii) all candidates who were entered for Latin and Greek while they were not entered for English were not entered for French;

(iv) all candidates who were entered for Latin and Greek while they were not entered for French were not entered for German.

Shew that,—

(1) Every candidate was either entered for English or else for both Latin and Greek.

(2) Every candidate was entered either for Latin or else for both English and French.

(3) All candidates entered for French were entered also for English.

(4)   All candidates entered for German were also entered for both English and French.

(5)   If a candidate was not entered for English, he was not entered for either French or German, but he was entered for both Latin and Greek.

(6)   If a candidate was not entered for French, he was entered for both Latin and Greek but not for German.

(7)   If a candidate was entered for Latin and also either entered for German or not entered for Greek, he was entered for English, French, and German.

(8)   If a candidate was entered both for Greek and German, he was also entered for English, Latin, and French.

(9)   If a candidate was entered neither for Greek nor German, he was entered for English and French but not for Latin.

(10)   Every candidate was entered for at least two languages; and no candidate who was entered for only two languages was entered for German.

394.   In a certain year it was observed that all horses entered for the One Thousand were also entered for the Oaks; all horses entered for the Two Thousand were also entered both for the Derby and the St Leger; all horses entered both for the One Thousand and the Derby were entered for the Two Thousand; all horses entered for the One Thousand and the St Leger were also entered for the Derby; all horses entered either for the Oaks or the St Leger were entered either for the Two Thousand or the One

Thousand. Shew that in that year all horses must belong to one or other of the following four classes :—

(1) Horses entered for all the following races,— the Two Thousand, the Derby, the Oaks, the St Leger.

(2) Horses entered for none of the following races,—the Two Thousand, the One Thousand, the Oaks, the St Leger.

(3) Horses entered for the Two Thousand, the Derby, and the St Leger, but not for the One Thousand.

(4) Horses entered for the One Thousand and the Oaks, but for none of the three other races.

395. Shew the equivalence between the three following sets of propositions :—

> (i) A is D ; (1)
> aB is Cde ; (2)
> Ce is A or B ; (3)
> ab is C ; (4)
> CDE is Ab ; (5)
> De is ABC or Abc ; (6)
> $AbcD$ is e. (7)

> (ii) CD is ABe or AbE ; (8)
> cD is A ; (9)
> d is $aBCe$ or $abCE$; (10)
> AcD is BE or be. (11)

> (iii) C is AD or ad ; (12)
> E is bC or $ABcD$; (13)
> e is BC or $AbcD$. (14)

To establish the desired result it will suffice to shew that (ii) may be inferred from (i), (iii) from (ii), and (i) from (iii).

First, (ii) may be inferred from (i).

By (5), CDE is Ab;
therefore, CDE is AbE.
By (6), CDe is AB;
therefore, CDe is ABe.
But, CD is CDE or CDe;
therefore, CD is AbE or ABe.   (8)

By (2), aB is C;
by (4), ab is C;
therefore, a is C;
therefore, c is A;
therefore, cD is A.   (9)

By (1), d is a;
therefore, d is aB or ab;
by (2), aB is $aBCe$;
by (3) and (4), ab is $abCE$;
therefore, d is $aBCe$ or $abCE$.   (10)

By (6), AcD is bc or E;
by (7), AcD is B or e;
therefore, AcD is BE or be.   (11)

Secondly, (iii) may be inferred from (ii).

By (8), CD is AD;
by (9), Cd is ad;
therefore, C is AD or ad.   (12)

By (8), E is Ab or c or d;
by (9), E is A or C or d;
therefore, E is Ab or d or Ac;
by (10), E is abC or D;
therefore, E is AbD or $abCd$ or AcD;

by (11), E is B or a or C or d;

therefore, E is $AbCD$ or $abCd$ or $ABcD$;

therefore, E is bC or $ABcD$. (13)

By (8), e is AB or c or d;

by (9), e is A or C or d;

therefore, e is AB or Ac or d;

by (10), e is aBC or D;

therefore, e is ABD or AcD or $aBCd$;

by (11), e is b or a or C or d;

therefore, e is $ABCD$ or $AbcD$ or $aBCd$;

therefore, e is BC or $AbcD$. (14)

Thirdly, (i) may be inferred from (iii).

By (13), Ec is D;

by (14), ec is D;

therefore, c is D;

therefore, Ac is D;

by (12), AC is D;

therefore, A is D. (1)

By (13), aB is e;

by (14), Be is C;

by (12), aC is d;

therefore, aB is Cde. (2)

By (14), Ce is B;

therefore, Ce is A or B. (3)

By (14), ab is E;

by (13), bE is C;

therefore, ab is C. (4)

By (12), CD is A;

by (13), CE is b;

therefore, CDE is Ab. (5)

By (14), *De* is *BCD* or *Abc*;
   by (12), *BCD* is *ABC*;
   therefore, *De* is *ABC* or *Abc*.   (6)
         By (13), *bc* is *e*;
         therefore, *AbcD* is *e*.   (7)

The desired conclusion might also be reached by shewing that each set of propositions may be summed up in the single proposition,—

Everything is *ABCDe* or *ABcDE* or *AbCDE* or *AbcDe* or
aBCde or *abCdE*.

396. Shew the equivalence between the two following sets of propositions :—

   (1) *A* is *BC* or *BE* or *CE* or *D* ;
        B is *ACDE* or *ACde* or *cdE* ;
        C is *AB* or *AE* or *aD* ;
        D is *ABCE* or *Ace* or *aC* ;
        E is *AC* or *aCD* or *Bc*.

   (2) *a* is *BcdE* or *bcde* or *bD* ;
        b is *a* or *ce* or *dE* ;
        c is *AbDe* or *abde* or *BdE* ;
        d is *abce* or *BcE* or *Be* or *bE* ;
        e is *ab* or *bc* or *d*.

397. Shew that the following sets of propositions are equivalent :—

   (1) *A* is *BCDE* or *BCd* or *bCD* or *bdE* ;
        C is *ABde* or *AbDe* or *aBE* or *BDE* ;
        E is *Abcd* or *aBC* or *BCD*.

(2) $AbCD$ is e;
    ACd is Be;
    ab is c;
    BCe is Ad;
    c is $AbdE$ or ae.

(3) $ABCd$ is e;
    ABd is C;
    AbC is D;
    $Abcd$ is E;
    ad is BCE or ce;
    D is $AbCe$ or ace or BCE.

398. Shew the equivalence between the two following sets of propositions :—

(1) a is $Bcde$ or $bcdE$;
    b is $Acde$ or $acdE$;
    C is DE;
    cE is abd;
    D is CE.

(2) AB is CDE or cde;
    ab is cdE;
    C is $ABDE$;
    D is $ABCE$;
    E is $ABCD$ or $abcd$.

399. Find which of the following propositions may be omitted without any limitation of the information given :—

Pq is rST;
Pr is qST;

Ps is QRt ;
Pt is QRs ;
pT is qr ;
Q is R ;
Qs is PRt ;
qS is rT ;
qT is PrS or pr ;
R is Q ;
Rs is PQt ;
rS is qT ;
rT is PqS or pq ;
st is PQR or pqr.

400. Given,—(i) AB is CD or Ef,

(ii) D is $AbeF$,

(iii) d is A or bc,

(iv) dE is ABC ;

then,—

(1) B is $ACdEf$,

(2) a is $bcde$,

(3) E is $ABCdf$,

(4) C is Ab or $AdEf$,

(5) bf is de,

(6) c is Abe or $abde$,

(7) A is $BCdEf$ or $bDeF$ or bde,

(8) e is Abd or $abcd$ or $AbDF$,

(9) F is $abcde$ or Abe,

(10) f is $Abde$ or $ABCdE$ or $abcde$.

(1) By (ii), B is d ;

by (iii), Bd is A ;

therefore, B is Ad ;

by (i), ABd is Ef;
therefore, B is $AdEf$;
by (iv), dE is C;
therefore, B is $ACdEf$.

(2)

By (ii), a is d;
by (iii), ad is bc;
therefore, a is bcd;
by (iv), bd is e;
therefore, a is $bcde$.

(3)

By (ii), E is d;
by (iv), dE is ABC;
therefore, E is $ABCd$;
by (i), ABd is f;
therefore, E is $ABCdf$.

(4)

By (1), C is b or $AdEf$;
by (2), C is A;
therefore, C is Ab or $AdEf$.

(5)

By (ii), f is d;
by (3), b is e;
therefore, bf is de.

(6)

By (ii), c is A or d;
therefore, c is A or ad;
by (1), c is b;
by (3), c is e;
therefore, c is Abe or $abde$.

(7)

By (1), B is $ACdEf$;
by (ii), D is $AbeF$;
by (iv), bd is e.

But, (by the law of excluded middle), A is B or D or bd;
therefore, A is $BCdEf$ or $bDeF$ or bde.

K. L.

(8)   By (2), a is bcd;
                   by (ii), D is AbF.

But, e is Ad or a or D;
therefore, e is Ad or $abcd$ or $AbDF$;
            and by (1), e is b;
therefore, e is Abd or $abcd$ or $AbDF$.

(9)   By (2), a is $abcde$;
               therefore, F is $abcde$ or A.

By (1) and (3), F is be;
   therefore, F is $abcde$ or Abe.

(10)   By (3), b is e;
                  by (1), B is $ACdE$;
                  by (2), a is $abcde$.

But, f is Ab or B or a;
therefore, f is Abe or $ABCdE$ or $abcde$;
            and by (ii), f is d;
therefore, f is $Abde$ or $ABCdE$ or $abcde$.

401. Three persons A, B, C, are set to sort a heap of books in a library. A is told to collect all the English political works, and the bound foreign novels: B is to take the bound political works, and the English novels, provided they are not political: to C are assigned the bound English works and the unbound political novels. What works will be claimed by two of them? Will any be claimed by all three?

[Venn, *Symbolic Logic*, pp. 264, 265.]

Let P = English,
   Q = political,
   R = bound,
   S = novel;

A = books assigned to A,

B = books assigned to B,

C = books assigned to C.

The premisses tell us that,—

(1) PQ is A,

(2) pRS is A,

(3) QR is B,

(4) PqS is B,

(5) PR is C,

(6) QrS is C;

and the problem is to determine of what we can affirm respectively AB, BC, CA, ABC.

By (1), what is both P and Q is A,

by (3), what is both Q and R is B;

therefore, what is both P, Q, and R is AB. (i).

Combining (2) and (3) similarly, we have

$pQRS$ is AB. (ii).

From (1) and (4) we get nothing, since nothing can be P and Q, and at the same time P and not-Q. Nor does the combination of (2) and (4) yield anything. We find then that " English bound political works and foreign bound political novels are claimed both by A and B."

Similarly, PQR is BC. (iii)

$PqRS$ is BC. (iv)

PQR is CA. (v)

$PQrS$ is CA. (vi)

Lastly, (i) and (iii) give PQR is ABC; and it will easily be seen that PQR is the only combination of which this is true.

402. (1) Where A or C or E is present, B or D or F is present, and *vice versâ;*

(2) Where B is present and C absent, or B absent and C present, D is present and E absent or D is absent and E present, and *vice versâ;*

(3) Where both A and D are present, F is absent;

(4) Where D is present, E is absent, and *vice versâ;*

(5) Where C is present, D is absent.

Shew that where C is present, B is absent, and *vice versâ.*   [Jevons, *Pure Logic*, pp. 66, 67.]

By (2), $\begin{cases} De \text{ is } Bc \text{ or } bC, \\ dE \text{ is } Bc \text{ or } bC; \end{cases}$

but by (4), Everything is De or dE;

therefore, Everything is Bc or bC;

therefore, $\begin{cases} C \text{ is } b, \\ b \text{ is } C. \end{cases}$

403. Given the same premisses as in the preceding example, shew that,—

(1) Where C and D are both absent, B and E are both present, and *vice versâ;*

(2) Where D is present, A and B are present, while C, E, and F are absent, and *vice versâ;*

(3) Where E is absent, A, B, and D are present, while C and F are absent, and *vice versâ;*

(4) Where B is absent, C, E, and F are present, while D is absent;

(5) Where D and F are both absent, B and E are present while C is absent.

[Jevons, *Pure Logic*, pp. 66, 67.]

404. With respect to certain classes of pheno-
mena, it is observed that,—

(i)   Where B is absent, E is present, but C, D, and
F are absent;

(ii) Where B is present while D is absent, A, C,
and E are present, but F is absent;

(iii) If B and D are both present, E is not present
F being absent, nor is C present A being absent.

It may hence be deduced that,—

(1) If A is absent, C is absent.

(2) If B is present, either C or D is present.

(3) If B, D, and E are all present, F is present.

(4) If C is absent, B and D are either both pre-
sent or both absent.

(5) If C and D are both absent, B is absent.

(6) If C is present, A and B are both present.

(7) If D is present, B is present.

(8) If D is absent, E is present but F is absent.

(9) If F is absent, D and E cannot both be pre-
sent or both absent.

405. Given,—

(1) aB is c or D;

(2) BE is DF or cdF;

(3) C is aB or BE or D;

(4) bD is e or F;

(5) bf is a or C or DE;

(6) $bcdE$ is Af or aF;

(7) A is B or $CDEf$ or cDf or cdE;

it follows that,—

> (i) A is B ;
> (ii) C is D ;
> (iii) E is F.

406.   One season at a certain hotel in Switzerland it happened that all the visitors were either English or Americans; all who went on mountaineering expeditions were either lawyers or English members of a University or unmarried American ladies; none of the lawyers were ladies; all the English lawyers were members of a University; all the ladies who were members of a University were American or unmarried ; all the Americans who were not members of a University were married ; all the members of a University who were not lawyers were mountaineers; the mountaineers who were members of a University were either Americans who were not lawyers or else ladies.

Obtain the fullest descriptions you can of the English mountaineers; the lawyers; the members of a University; those who were not members of a University ; the American ladies ; the American mountaineers ; the unmarried non-mountaineers ; the unmarried men ; the married men who were not lawyers.

407.   Shew the equivalence between the two following sets of propositions :—

> (1) AB is CD or EF;
> Cd is Ab or Ef;

eF is aB or cD;
ab is cd;
cd is ef;
ef is ab.

(2) a is BC or BD or $bcdef$;
AB is CDE or $cDEF$;
e is $abcdf$ or F;
$Abcd$ is ef;
$aBCd$ is f;
$AbCF$ is E.

408. Given,—

(1) bc is DE or Df or hi,
(2) C is aB or $DEFG$ or BFH,
(3) Bcd is eK or hi,
(4) Acf is d,
(5) i is BC or Cd or Cf or H,
(6) $ABCDEFG$ is H or I,
(7) $DEFGH$ is B,
(8) ABk is f or h,
(9) $ADFIk$ is H,
(10) $ADEFH$ is B or C or G or K;

shew that,—A is K.

This problem involves ten terms; and its solution will shew the power of the methods that we have been considering.

It may be solved in a straightforward manner by the general method formulated in section 355 :—

By (1), A is B or C or DE or Df or hi;

By (2), A is BFH or c or $DEFG$;

therefore, A is Bc or BFH or cDE or cDf or chi or $DEFG$;

By (3), A is b or C or D or hi or K;
therefore, A is $BCFH$ or BcD or $BDFH$ or cDE or cDf
or chi or $DEFG$ or K;

By (4), A is C or d or F;
therefore, A is $BCFH$ or $BcDF$ or $BDFH$ or $cDEF$
or $cdhi$ or $cFhi$ or $DEFG$ or K;

By (5), A is BC or Cd or Cf or H or I;
therefore, A is $BCDEFG$ or $BCFH$ or $BcDFI$ or $BDFH$
or $cDEFH$ or $cDEFI$ or $DEFGH$ or $DEFGI$ or K;

By (6), A is b or c or d or e or f or g or H or I;
therefore, A is $BCFH$ or $BcDFI$ or $BDFH$ or $cDEFH$
or $cDEFI$ or $DEFGH$ or $DEFGI$ or K;

By (7), A is B or d or e or f or g or h;
therefore, A is $BCFH$ or $BcDFI$ or $BDEFGI$ or $BDFH$
or $cDEFgH$ or $cDEFgI$ or $cDEFhI$ or $DEFGhI$ or K;

By (8), A is b or f or h or K;
therefore, A is $BcDFhI$ or $bcDEFgH$ or $bcDEFgI$
or $cDEFhI$ or $DEFGhI$ or K;

By (9), A is d or f or H or i or K;
therefore, A is $bcDEFgH$ or K;

By (10), A is B or C or d or e or f or G or h or K;
therefore, A is K.

The problem may also be solved as follows :—

By (9), ADF is H or K or i;

By (6), $ABCDEFG$ is H or I;
therefore, $ABCDEFG$ is H or K;

By (8), ABF is h or K;
therefore, $ABCDEFG$ is K;
therefore, No $ABCDEFG$ is k;
therefore, No $ABCk$ is $DEFG$.   (i)

By (2), AC is $DEFG$ or BFH;

But by (8), BFH is a or K;

therefore, ACk is $DEFG$;

therefore, by (i), No ACk is $ABCk$;

therefore, No AC is Bk;

therefore, ABC is K.   (ii)

By (3), Bcd is K or hi;

But by (5), hi is C;

therefore, Bcd is K.   (iii)

By (9), $ADFk$ is H or i;

By (5), c is H or I;

therefore, $AcDF$ is H or K;

By (8), ABF is h or K;

therefore, $ABcDF$ is K;

therefore, $ABcD$ is f or K;

By (4), Acf is d;

therefore, AcD is F;

therefore, $ABcD$ is K;

But by (iii), $ABcd$ is K;

therefore, ABc is K;

And by (ii), ABC is K;

therefore, AB is K.   (iv)

By (5), hi is BC or Cd or Cf;

therefore, bDF is H or I;

By (9), ADF is H or i or K;

therefore, $AbDF$ is H or K.   (v)

By (7), $bDEF$ is g or h;

therefore, $AbDEF$ is g or K;

therefore, $AbDEFG$ is K;

therefore, $DEFG$ is a or B or K;

By (2), AbC is $DEFG$;

therefore, AbC is K.   (vi)

By (10), *AbcDEF* is *G* or *h* or *K*;

By (7), *bDEF* is *g* or *h*;

therefore, *AbcDEF* is *h* or *K*;

By (v), *AbDF* is *H* or *K*;

therefore, *AbcDEF* is *K*;

therefore, *AbcDE* is *f* or *K*;

By (4), *AcD* is *F*;

therefore, *AbcDE* is *K*.   (vii)

By (1), *bc* is *DE* or *Df* or *hi*;

and by (5), *bc* is *H* or *I*;

therefore, *bc* is *DE* or *Df*;

By (4), *Ac* is *d* or *F*;

therefore, *Abc* is *DE*;

therefore, *Abc* is *AbcDE*;

and therefore, by (vii), *Abc* is *K*;

But by (vi), *AbC* is *K*;

therefore, *Ab* is *K*;

and by (iv), *AB* is *K*;

therefore, *A* is *K*.

CHAPTER XII.

INVERSE PROBLEMS.

409. Nature of the Inverse Problem.

By the *Inverse Problem* I mean a certain problem so-called by Professor Jevons. Its nature will be indicated by the following extracts, which are from the *Principles of Science* and the *Studies in Deductive Logic* respectively.

"In the Indirect process of Inference we found that from certain propositions we could infallibly determine the combinations of terms agreeing with those premisses. The inductive problem is just the inverse. Having given certain combinations of terms, we need to ascertain the propositions with which they are consistent, and from which they may have proceeded. Now if the reader contemplates the following combinations,—

$$ABC \qquad abC$$
$$aBC \qquad abc,$$

he will probably remember at once that they belong to the premisses $A = AB$, $B = BC$. If not, he will require a few trials before he meets with the right answer, and every trial

will consist in assuming certain laws and observing whether
the deduced results agree with the data. To test the facility
with which he can solve this inductive problem, let him
casually strike out any of the possible combinations involving
three terms, and say what laws the remaining combinations
obey. Let him say, for instance, what laws are embodied
in the combinations,—

$$ABC \qquad aBC$$
$$Abc \qquad abC.$$

" The difficulty becomes much greater when more terms
enter into the combinations. It would be no easy matter
to point out the complete conditions fulfilled in the com-
binations,—

$$ACe$$
$$aBCe$$
$$aBcdE$$
$$abCe$$
$$abcE.$$

After some trouble the reader may discover that the prin-
cipal laws are $C = e$, and $A = Ae$; but he would hardly
discover the remaining law, namely that $BD = BDc$."
(*Principles of Science*, 1st ed., vol. I., p. 144; 2nd ed., p. 125.)

"The inverse problem is always tentative, and consists
in inventing laws, and trying whether their results agree
with those before us." (*Studies in Deductive Logic*, p. 252.)

I should myself rather prefer to state the problem as
follows :—

Given a single proposition of the form,—

Everything is P_1 or P_2......or P_n;

*to find a set of propositions involving as simple relations as
possible which shall be equivalent to it.*

It is strictly true that the inverse problem is indetermi-
nate, for since we may find a number of sets of propositions
which are precisely equivalent in logical force, any inverse
problem will admit of a number of solutions.   But I do not
think that it is necessary in order to solve any inverse pro-
blem to have recourse to a series of guesses, nor that the
method of solution need be described as tentative.   In the
following section, I give what appears to be an easy rule
for finding a fairly satisfactory solution of any inverse
problem.   Since, however, a number of solutions are possi-
ble, some of which are simpler than others, the process
must be regarded as tentative so far as we seek to obtain
the most satisfactory solution.

We can hardly lay down any absolute standard of sim-
plicity; but comparing two equivalent sets of propositions,
we may generally speaking regard that one as the simpler
which contains the smaller number of categorical propo-
sitions[1].   If the number of such propositions is equal, then
I should count the number of terms involved in their sub-
jects and predicates taken together, and regard that one as
the simpler which involves the fewer terms.   If we have to
compare disjunctives with categoricals, we may regard a
proposition with two alternatives in the predicate as equiva-
lent to two categorical propositions, one with three alterna-
tives as equivalent to three categorical propositions, and
so on[2].

[1] By a categorical proposition here I mean one which does not
involve disjunctive combination, (although it may involve conjunctive
combination), either in the subject or in the predicate.
[2] When Professor Jevons speaks of the extreme difficulty of the
inverse process, he apparently has in view a resolution into a small
number of categorical propositions; and at this I have aimed in my
solutions of inverse problems.

410.   A General Solution of the Inverse Problem,
—*i.e.*, Given a proposition limiting us to a number of
complex alternatives to find a set of propositions in-
volving as simple relations as possible which shall be
equivalent to it.

The data may be written in the form,—

Everything is P or Q or S or T or &c.,
where P, Q, &c., are complex terms.

By contraposition[1] we may bring over one or more of
these complex terms from the predicate into the subject,
so that we have,—

What is neither P nor S nor &c. is Q or T or &c.

The selection of certain terms for transposition in this
way is arbitrary, (and it is here that the indeterminate-
ness of the problem becomes apparent); but it will gene-
rally be found best to take two or three which have as much
in common as possible.

"What is neither P nor S nor &c. is Q or T or &c."
will immediately resolve itself into a series of propositions,
which taken together give all the information originally
given[2]. If any of these are themselves very complex we
may proceed with them in the same way. We may then
suppose ourselves left with a series of fairly simple propo-
sitions ; but it will probably be found that some of these
merely repeat information given by others, so that they
may be omitted.

We may find to what extent this is the case, by adopting
any one of the three following methods :—

First, by leaving out each proposition in turn, and de-
termining (by the ordinary rules) what the remainder by
combination give concerning its subject. If we find that

---

[1] Cf. section 317.   [2] Cf. chapter II.

it adds nothing to the information that they give it may be omitted.

Secondly, by bringing each proposition to the form,—

Nothing is X_1 or X_2............or X_n,

and then comparing it with the combination of the remainder.

Thirdly, by writing down all possible combinations after Jevons's plan, (*Pure Logic*, p. 46; *Principles of Science*, Chapter VI; *Studies*, p. 181), and noting which are excluded by each proposition in turn. If a proposition excludes no combination that is not also excluded by other propositions it may be omitted.

We are now left with a series of propositions which are mutually independent. By further comparison however we shall probably find that some of them may be still further simplified. When such simplification has been carried as far as possible we shall have our final solution.

This may be *verified* by recombining the propositions that we have obtained, by which operation we ought to arrive again at the series of alternatives with which we started.

To illustrate the above method, four examples follow which are worked out in full detail.

I.   For our first example we may take one of those chosen by Jevons in the extract quoted in section 409.

Given the proposition that " Everything is

$$ABC$$
$$\text{or } Abc$$
$$\text{or } aBC$$
$$\text{or } abC,"$$

find a set of propositions involving as simple relations as possible which shall be equivalent to it.

400 COMPLEX INFERENCES. [PART IV.

By contraposition, What is neither ABC nor Abc is aBC or abC; therefore, What is a or Bc or bC is aBC or abC;

$$i.e., \quad \begin{cases} a \text{ is } C, \\ Bc \text{ is not,} \\ bC \text{ is } a. \end{cases}$$

"Bc is not" is reducible to "B is C"; and this proposition and "a is C" may be combined into "c is Ab."

Our solution therefore is,—

$$\begin{cases} c \text{ is } Ab, \\ bC \text{ is } a. \end{cases}$$

By combining these propositions it will be found that we regain the original proposition.

II. We may next take the more complex example contained in the extract from Jevons quoted in section 409.

The given alternatives are,—

$$\begin{array}{c} ACe, \\ aBCe, \\ aBcdE, \\ abCe, \\ abcE. \end{array}$$

Therefore, What is not $aBcdE$ or $abcE$ is ACe or $aBCe$ or $abCe$;

therefore, $\begin{cases} A \text{ is } Ce; \\ C \text{ is } Ae \text{ or } aBe \text{ or } abe; \\ e \text{ is } AC \text{ or } aBC \text{ or } abC; \\ BD \text{ is } ACe \text{ or } aCe; \end{cases}$

these propositions are immediately reducible to,—

$$\begin{cases} A \text{ is } Ce; \\ C \text{ is } e; \\ e \text{ is } C; \\ BD \text{ is } Ce; \end{cases}$$

and they may be further resolved into,—

$$\begin{cases} E \text{ is } ac; \\ c \text{ is } aE; \\ BD \text{ is } Ce. \end{cases}$$

This solution again may be verified by re-combination.

III.   The following problem is from Jevons, *Principles of Science*, 2nd ed., p. 127, (Problem v.).

The given alternatives are,—

$$ABCD,$$
$$ABCd,$$
$$ABcd,$$
$$AbCD,$$
$$AbcD,$$
$$aBCD,$$
$$aBcD,$$
$$aBcd,$$
$$abCd.$$

Then, by contraposition, what is neither of the four following,—

$$ABCD,$$
$$ABCd,$$
$$aBcD,$$
$$aBcd,$$

must be one of the remainder.

But " What is neither *ABCD, ABCd, aBcD* nor *aBcd*," is equivalent to " What is neither *ABC* nor *aBc*," and that is equivalent to " What is *b* or *Ac* or *aC*."

Therefore, What is *b* or *Ac* or *aC* is *ABcd* or *AbCD* or *AbcD* or *aBCD* or *abCd*.

From this we have our first resolution of the given information in the three propositions:—

(1) b is AD or aCd;

(2) Ac is Bd or bD;

(3) aC is BD or bd.

Each of these may again be broken up into two propositions :—

$$b \text{ is } AD \text{ or } aCd,$$

becomes   If b is not AD, it is aCd;

that is,   (i) ab is Cd,

                  (ii) bd is aC.

(2) may similarly be broken up into,—

                  (iii) ABc is d,

                  (iv) Acd is B;

and (3) into,—   (v) aBC is D,

                  (vi) aCD is B.

But (iv) is inferrible from (ii), and (vi) is inferrible from (i); (iv) and (vi) may therefore be omitted.

We have then for our final solution,—

                  (1) ab is Cd,

                  (2) bd is aC,

                  (3) ABc is d,

                  (4) aBC is D.

This is practically equivalent to the solution given in Jevons, *Studies*, p. 256.

We may now verify it as follows:—

By (1),   Everything is A or B or Cd;

By (2),   Everything is aC or B or D;

therefore,   Everything is AD or aCd or B;

By (3),   Everything is a or b or C or d;
therefore, Everything is AbD or ACD or aB or aCd or BC
or Bd;

By (4),   Everything is A or b or c or D;
therefore, Everything is ABC or ABd or AbD or ACD or
aBc or aBD or $abCd$ or BCD or Bcd.

But, AbD is $AbCD$ or $AbcD$.

Expanding all the terms similarly, we have,—

Everything is $ABCD$,
or $ABCd$,
or $ABcd$,
or $AbCD$,
or $AbcD$,
or $aBCD$,
or $aBcD$,
or $aBcd$,
or $abCd$.

These are precisely the alternatives that were originally
given us.

IV.   The following example is also from Jevons, *Principles of Science*, 2nd Edition, p. 127, (Problem viii).   In his
Studies, p. 256, he speaks of the solution as *unknown ;* and
I am, therefore, the more interested in shewing that a fairly
simple solution, involving no more than *five* categorical
propositions, may be obtained by the application of the
general rule formulated in this section.

The given alternatives are,—

$ABCDE$,
$ABCDe$,
$ABCde$,
$ABcde$,

COMPLEX INFERENCES.   [PART IV.

$$AbCDE,$$
$$AbcdE,$$
$$Abcde,$$
$$aBCDe,$$
$$aBCde,$$
$$aBcDe,$$
$$abCDe,$$
$$abCdE,$$
$$abcDe,$$
$$abcdE.$$

Therefore, What is neither $ABCDE$ nor $ABCDe$ nor $ABCde$ nor $Abcde$ nor $aBCDe$ nor $aBCde$ is $ABcde$ or $AbCDE$ or $AbcdE$ or $aBcDe$ or $abCDe$ or $abCdE$ or $abcDe$ or $abcdE$.

Now, "What is neither $ABCDE$ nor $ABCDe$ nor $ABCde$ nor $Abcde$ nor $aBCDe$ nor $aBCde$" is equivalent to "What is either of the following,—

$$dE,$$
$$bC,$$
$$bD,$$
$$bE,$$
$$Bc,$$
$$cD,$$
$$cE,$$
$$aE,$$
$$ab,$$
$$ac."$$

Moreover, bE is either bD or dE;
cE is either cD or dE;
ab is either aE or abe;
ac is either aE or ace;
bD is either bC or cD.

Therefore, our proposition becomes " What is

$$dE,$$
$$\text{or } bC,$$
$$\text{or } Bc,$$
$$\text{or } cD,$$
$$\text{or } aE,$$
$$\text{or } abe,$$
$$\text{or } ace,$$

is either
$$ABcde,$$
$$\text{or } AbCDE,$$
$$\text{or } AbcdE,$$
$$\text{or } aBcDe,$$
$$\text{or } abCDe,$$
$$\text{or } abCdE,$$
$$\text{or } abcDe,$$
$$\text{or } abcdE$$ ";

and this is resolvable into the following set of propositions,—

(1) dE is ab or bc ;
(2) bC is ADE or aDe or adE ;
(3) Bc is Ade or aDe ;
(4) cD is ae ;
(5) aE is bd ;
(6) abe is D ;
(7) ace is D.

Of these, (2) may be broken up into,—

(8) AbC is DE ;
(9) $bCDE$ is A ;
(10) $bCde$ is non-existent.

But (9) may be inferred from (5), and (10) may be inferred from (6) and (8); (8) may therefore be substituted for (2).

(3) may be inferred from (1), (4), and (7), and may therefore be omitted.

(1) may be broken up into,—

> (11) *AdE* is *bc*;
> (12) *BdE* is non-existent.

But (12) may be inferred from (5) and (11); (11) may therefore be substituted for (1).

Again, (6) and (7) may be combined into,—

> (13) *ade* is *BC*,

which may therefore be substituted for them.

Our set of propositions may therefore be reduced to,—

> (i) *AdE* is *bc*;
> (ii) *AbC* is *DE*;
> (iii) *cD* is *ae*;
> (iv) *aE* is *bd*;
> (v) *ade* is *BC*.

From (iv) it follows that *acD* is *e*; (iii) may therefore be reduced to *cD* is *a*.

From (ii) it follows that *AbdE* is *c*; (i) may therefore be reduced to *AdE* is *b*.

We are, therefore, left with the following as our final solution:—

> (i) *AdE* is *b*;
> (ii) *AbC* is *DE*;
> (iii) *cD* is *a*;
> (iv) *aE* is *bd*;
> (v) *ade* is *BC*.

This solution may be verified as follows:—

> By (i), Everything is *a* or *b* or *D* or *e*;

By (ii), Everything is a or B or c or DE;

therefore, Everything is a or BD or Be or bc or cD
or ce or DE;

By (iii), Everything is a or C or d;

therefore, Everything is a or BCD or BCe or Bde or bcd
or CDE or cde;

By (iv), Everything is A or bd or e;

therefore, Everything is $ABCD$ or $ACDE$ or abd or ae
or BCe or Bde or bcd or cde;

By (v), Everything is A or BC or D or E;

therefore, Everything is $ABCD$
or $ABde$
or $Abcd$
or $ACDE$
or $Acde$
or $abdE$
or aDe
or BCe
or $bcdE$.

But $ABCD$ is $ABCDE$ or $ABCDe$, and so with the others.

Expanding the terms in this way, we have,—

Everything is $ABCDE$
or $ABCDe$
or $ABCde$
or $ABcde$
or $AbCDE$
or $AbcdE$
or $Abcde$
or $aBCDe$

> or *aBCde*
> or *aBcDe*
> or *abCDe*
> or *abCdE*
> or *abcDe*
> or *abcdE*.

These are again the alternatives with which we commenced.

411. Another Method of Solution of the Inverse Problem.

Another method of solving the Inverse Problem, suggested to me (in a slightly different form) by Mr Venn, is to write down the original complex proposition in the negative form, *i. e.*, to obvert it, before resolving it. It has already been shewn that a negative proposition with a disjunctive predicate, may be immediately broken up into a set of simpler propositions.

In some cases, especially where the number of destroyed combinations as compared with those that are saved is small, this plan is of easier application than that given in the preceding section.

To illustrate this method we may take two or three of the examples already discussed.

I.   Everything is *ABC* or *Abc* or *aBC* or *abC*;

therefore, by obversion, Nothing is *AbC* or *Bc* or *ac*;

and this proposition is at once resolvable into,—

$$\begin{cases} Ab \text{ is } c, \\ c \text{ is } Ab\,[1]. \end{cases}$$

---

[1] The student will immediately recognize that this is equivalent to our former solution.   Equationally it would be written $Ab = c$.

II. Everything is ACe or $aBCe$ or $aBcdE$ or $abCe$ or $abcE$;

therefore, by obversion, Nothing is AE or CE or BDE or Ac or BcD or ce.

This proposition may be successively resolved as follows :—

$$\begin{cases} \text{No } E \text{ is } A \text{ or } C \text{ or } BD; \\ \text{No } c \text{ is } A \text{ or } BD \text{ or } e. \end{cases}$$

$$\begin{cases} E \text{ is } ac\,; \\ E \text{ is } b \text{ or } d\,; \\ c \text{ is } aE; \\ c \text{ is } b \text{ or } d. \end{cases}$$

$$\begin{cases} E \text{ is } ac; \\ BD \text{ is } Ce\,; \\ c \text{ is } aE. \end{cases}$$

This is the same solution that we reached before.

III. Everything is $ABCD$ or $ABCd$ or $ABcd$ or $AbCD$ or $AbcD$ or $aBCD$ or $aBcD$ or $aBcd$ or $abCd$;

therefore, by obversion, Nothing is Abd or bcd or $ABcD$ or abc or abD or $aBCd$;

and this proposition may be successively resolved as follows :—

$$\begin{cases} \text{No } bd \text{ is } A \text{ or } c\,; \\ \text{No } ABc \text{ is } D\,; \\ \text{No } ab \text{ is } c \text{ or } D\,; \\ \text{No } aBC \text{ is } d. \end{cases}$$

$$\begin{cases} bd \text{ is } aC\,; \\ ABc \text{ is } d\,; \\ ab \text{ is } Cd\,; \\ aBC \text{ is } D. \end{cases}$$

This again repeats our original solution. It is curious that in each of the above cases we should by independent methods have attained the same result.

412. It is observed that the phenomena A, B, C occur only in the combinations ABc, abC, and abc. What propositions will express the laws of relation between these phenomena ? [Jevons, *Studies*, p. 219.]

Everything is ABc or abC or abc. Noticing that "abC or abc" is equivalent to ab, we have by contraposition, What is not ab is ABc; that is, What is A or B is ABc;

$$\text{that is,} \quad \begin{cases} A \text{ is } Bc, \\ B \text{ is } Ac. \end{cases}$$

413. Find propositions that leave only the following combinations,—$ABCD$, $ABcD$, $AbCd$, $aBCd$, $abcd$.
                    [Jevons, *Studies*, p. 254.]

Jevons gives this as the most difficult of his series of inverse problems involving four terms. It may be solved as follows:—

Everything is $ABCD$ or $ABcD$ or $AbCd$ or $aBCd$ or $abcd$.

Noticing that $ABCD$ or $ABcD$ is equivalent to ABD, we have, What is neither $AbCd$ nor $aBCd$ is ABD or $abcd$.

Therefore, What is AB or ab or c or D is ABD or $abcd$; and this is resolvable into the four propositions,—

$$\begin{cases} AB \text{ is } D, \quad (1) \\ ab \text{ is } cd, \quad (2) \\ c \text{ is } ABD \text{ or } abd, \quad (3) \\ D \text{ is } AB. \quad (4) \end{cases}$$

But by (4) D is AB, and by (2) ab is d; therefore (3) may be reduced to c is D or ab,

$$\text{i.e., } cd \text{ is } ab.$$

Our set of propositions may therefore be reduced to,—

$$\begin{cases} AB \text{ is } D, \\ ab \text{ is } cd, \\ cd \text{ is } ab, \\ D \text{ is } AB\,^1. \end{cases}$$

414. It is observed that the phenomena A, B, C, D, E, F are present or absent only in the combinations,—$ABCDF$, $ABCDef$, $ABCdEf$, $ABcDF$, $ABcDef$, $aBcDF$, $aBcDef$, $bcdEf$. What propositions will express the laws of relation between these phenomena? [Jevons, *Studies*, p. 257.]

Jevons gives five solutions more or less differing from one another, and all expressed equationally. The following is still another solution expressed in the ordinary propositional forms :—

$$\begin{cases} BEf \text{ is } Cd, \\ b \text{ is } d, \\ C \text{ is } AB, \\ d \text{ is } Ef. \end{cases}$$

415. Resolve the proposition "Everything is one or other of the following,—

$$ABCDeF,$$
$$ABcDEf,$$
$$AbCDEF,$$
$$AbCDeF,$$
$$AbcDeF,$$

---

[1] Written equationally, this solution would appear still simpler; namely,—

$$AB = D,$$
$$ab = cd.$$

$$aBCDEf,$$
$$aBcDEf,$$
$$abCDeF.$$
$$abCdeF,$$
$$abcDef,$$
$$abcdef,"$$

into a series of simple propositions.

[Jevons, *Principles of Science*, 2nd ed., p. 127,

(Problem X.).]

The following is a solution :—

(1) ABE is cDf;
(2) $AcDF$ is be;
(3) aF is bCe;
(4) bf is ace;
(5) d is ae;
(6) ef is abc.

This is rather less complex than the solution by Dr John Hopkinson given in Jevons, *Studies*, p. 256, namely :—

(1) d is ab;
(2) b is AF or ae;
(3) Af is $BcDE$;
(4) E is Bf or $AbCDF$;
(5) Be is $ACDF$;
(6) abc is ef;
(7) $abef$ is c.

It will be a useful exercise for the student to shew that these two sets of propositions are really equivalent.

416. How many and what non-disjunctive propositions are equivalent to the statement that "What is either Ab or bC is Cd or cD, and *vice versâ*"?

[Jevons, *Studies*, p. 246.]

We have given,—

$$\begin{cases} Ab \text{ is } Cd \text{ or } cD, & (1) \\ bC \text{ is } Cd \text{ or } cD, & (2) \\ Cd \text{ is } Ab \text{ or } bC, & (3) \\ cD \text{ is } Ab \text{ or } bC. & (4) \end{cases}$$

(1) may be resolved into,—

$$\begin{cases} Abc \text{ is } D, & (5) \\ AbD \text{ is } c. & (6) \end{cases}$$

(2) becomes $\qquad bC \text{ is } d.$ (7)

(3) may be resolved into,—

$$\begin{cases} aCd \text{ is } b, & (8) \\ BC \text{ is } D. & (9) \end{cases}$$

(4) may be resolved into,—

$$\begin{cases} ac \text{ is } d, & (10) \\ Bc \text{ is } d. & (11) \end{cases}$$

But (6) may be inferred from (7); and (8) from (9).
We therefore have for our solution :—

$$\begin{cases} Abc \text{ is } D, \\ bC \text{ is } d, \\ BC \text{ is } D, \\ ac \text{ is } d, \\ Bc \text{ is } d. \end{cases}$$

417. The following is a further series of inverse
problems, which should be solved by the methods
indicated in sections 410 and 411.

In each case we have given a complex proposition
which it is desired to resolve into a series of relatively
simple propositions.

(1) Everything is $ABCD$ or $aBCD$ or $aBCd$ or
$abCd$ or $abcD$ or $abcd$.

(2) Everything is *ABCD* or *AbCd* or *aBcD* or *abcd*.

(3) Everything is *AbCD* or *AbCd* or *Abcd* or *aBcd* or *abCD* or *abCd* or *abcd*.

(4) Everything is *AbcDE* or *aBCd* or *aBCE* or *aBcd* or *aBde* or *abCe* or *abce* or *abDe* or *abde* or *BcdE* or *bCDe*.

(5) Everything is *ABCE* or *ABcd* or *ABcE* or *ABde* or *Abcd* or *abCE* or *abcE* or *abdE* or *abde* or *BCde*.

(6) Everything is *ABCDE* or *ABCdE* or *ABcDE* or *ABcDe* or *ABcde* or *AbCdE* or *Abcde* or *aBCDE* or *aBCde* or *abCDE* or *abcDe*.

(7) Everything is *ABDe* or *ABDF* or *AcDe* or *Acef* or *aBDe* or *aBDF* or *abCD* or *abCd* or *abcD* or *abcd* or *aCDE* or *aCDe* or *aCdE* or *aCde* or *acDe* or *aDEF* or *aDEf* or *aDeF* or *aDef* or *BcDF* or *bceF* or *bcef*.

(8) Everything is *AbdE* or *Abef* or *AbF* or *Acdef* or *aBDF* or *abCF* or *aCdE* or *ade* or *bCDe* or *bCdf* or *bDEF*.

(9) Everything is *ABCEf* or *Abe* or *aBCdf* or *aBcdE* or *aBcdeF* or *abef* or *bceF*.

(10) Everything is *ABcEF* or *ABDEF* or *AbCdef* or *Abcdef* or *AbcdF* or *AbD* or *AbdE* or *AbdeF* or *abCef* or *aBc* or *aBCd* or *aBCDe* or *aBCDEf* or *abdef*.